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Ternary logic control for stability modification of bilinear systems

In this work the problem of stability modification of bilinear control systems [1], [2]:

$$(1)\qquad d\boldsymbol{x}/dt = \boldsymbol{A}_0\boldsymbol{x} + u_1\boldsymbol{B}_1\boldsymbol{x} + \ldots + u_m\boldsymbol{B}_m\boldsymbol{x}, \quad |u_i(t)| < \alpha, \quad i = 1, 2, \ldots, m,$$

where $\boldsymbol{x} \in R^n$ denotes the state vector, α is a control bound, $\boldsymbol{A}_0$ is a Hurwitz $n \times n$ matrix and B_i, $i = 1, 2 \ldots, m$, are $n \times n$ matrices, is considered.

Applying a quadratic Lyapunov function $V_{\boldsymbol{S}}(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{S}\boldsymbol{x}$ such that the index of exponential stability

$$(2)\qquad \gamma_0(\boldsymbol{S}) = -\sup_{\boldsymbol{x}\neq\boldsymbol{0}}\left[\frac{\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{A}_0\boldsymbol{x}}{\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{x}}\right] > 0,$$

it is proved that optimal controls improving stability of system (1) are of the bang-bang form:

$$(3)\qquad \hat{u}_i = -\alpha\,\mathrm{sign}[\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{B}_i\boldsymbol{x}]; \quad i = 1, 2, \ldots, m,$$

with quadratic switching surfaces [2]. Moreover, stability properties of the closed-loop system at $\boldsymbol{x} = 0$ in the state space R^n are determined by the stability index

$$(4)\qquad \gamma(\boldsymbol{S}) = -\sup_{\boldsymbol{x}\neq\boldsymbol{0}}\left[\frac{\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{A}_0\boldsymbol{x}}{\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{x}} - \alpha\cdot\frac{|\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{B}_1\boldsymbol{x}|}{\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{x}} - \ldots - \alpha\cdot\frac{|\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{B}_m\boldsymbol{x}|}{\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{x}}\right],$$

which is always greater than or equal to $\gamma_0(\boldsymbol{S})$.

If an additional bound on the total "power" of controls is assumed

$$(5)\qquad |u_1| + \ldots + |u_m| \leqslant k\alpha, \quad k \in \{1, 2, \ldots, m\},$$

then at any state $\boldsymbol{x}(t)$ only k (out of m) controls $u_{i1}, \ldots, u_{ik}$ satisfying the inequality relation

$$(6)\qquad |\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{B}_{i1}\boldsymbol{x}| \geqslant \ldots \geqslant |\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{B}_{im}\boldsymbol{x}|$$

can be switched on, i.e. be non-zero [2]. Therefore the feedback controller has to make an appropriate choice of the active controls in real time according to the actual state $\boldsymbol{x}(t)$ of the system. Since the optimal controls are three-valued $(0, +\alpha, -\alpha)$ and the input information contained in inequalities (6) is also ternary, the controller is in fact a ternary logic system.

The general structure of the ternary logic controller of system (1) is proposed. It is proved that the controller can be realized by a multilayer neural network composed of the fundamental blocks: input layer, comparator, selector and converter.

References

[1] P. O. Gutman, *Stabilizing controllers for bilinear systems*, IEEE Transactions on Automatic Control AC-26 (1981), 917–922.
[2] A. Ossowski, *Analiza jakościowa w zagadnieniach dynamiki i sterowania układów mechatronicznych*, Prace IPPT 8 (2007).