Alicja Dembczak-Kołodziejczyk, Anna Lytova
University of Opole

On applications of random matrices in studies of the vibrational spectrum of glasses

Random matrices are widely used in physics, number theory, mathematical statistics and many other fields of mathematics, science, economics etc. In particular, recently in [1, 2] random matrices were used to construct models for the stiffness matrix and the Hessian matrix describing the vibrational spectrum of glasses. Inspired by [1, 2], we generalise the proposed models and, given $n, m \in \mathbb{N}$, consider two classes of large random matrices of the form

$$\mathcal{L}_n = \sum_{\alpha=1}^{m} \xi_\alpha \mathbf{y}_\alpha \mathbf{y}_\alpha^T \quad \text{and} \quad \mathcal{A}_n = \sum_{\alpha=1}^{m} \xi_\alpha (\mathbf{y}_\alpha \mathbf{x}_\alpha^T + \mathbf{x}_\alpha \mathbf{y}_\alpha^T),$$

where for every n, $(\xi_\alpha)_\alpha$ are iid copies of a random variable $\xi = \xi(n) \in \mathbb{R}$, $(\mathbf{x}_\alpha)_\alpha$, $(\mathbf{y}_\alpha)_\alpha \subset \mathbb{R}^n$ are two (not necessarily independent) sets of independent random vectors having *different covariance matrices* and generating well concentrated bilinear forms. We consider two main asymptotic regimes as $n, m(n) \to \infty$: a standard one, where $m/n \to c$, and a slightly modified one, where $m/n \to \infty$ and $\mathbf{E}\xi \to 0$ while $m\mathbf{E}\xi/n \to c$ for some $c \geqslant 0$. Assuming that vectors $(\mathbf{x}_\alpha)_\alpha$ and $(\mathbf{y}_\alpha)_\alpha$ are normalized and isotropic "in average", we prove the convergence in probability of the empirical spectral distributions of $\mathcal{L}_n$ and $\mathcal{A}_n$ to a version of the Marchenko–Pastur law and so called effective medium spectral distribution, correspondingly. In particular, choosing normalized Rademacher random variables as $(\xi_\alpha)_\alpha$, in the modified regime one can get a shifted semicircle and semicircle laws. This talk is based on [3].

References

[1] G. M. Cicuta, J. Krausser, R. Milkus, A. Zaccone, *Unifying model for random matrix theory in arbitrary space dimensions*, Phys. Rev. E 97 (2018), no. 3, art. 032113, 8 pp., MR3789138.

[2] M. Pernici, G. M. Cicuta, *Proof of a conjecture on the infinite dimension limit of a unifying model for random matrix theory*, J. Stat. Phys. 175 (2019), 384–401, MR3968860.

[3] A. Dembczak-Kołodziejczyk, A. Lytova, *On the empirical spectral distribution for certain models related to sample covariance matrices with different correlations*, Random Matrices Theory Appl. 11 (2022), no. 3, art. 2250030, 23 pp.; arXiv:2103.03204 (2021), MR4440256.