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Optimization problems for retrial systems with changeable service rate

We deal with a finite retrial queueing systems of the type $E/E/1$ in which customers arrive in a Poisson process with rate λ. The retrial time distribution is exponential with parameter ν. The service time distribution function is an exponential for both primary and repeated calls and the parameter of that distribution is defined in the following way. If at the instant a customer (primary or repeated) gets to service we have j customers in the system, then the parameter of service time is equal to μ_j. The number of places in the orbit is supposed to be equal to $m < \infty$.

Let $\xi(t)$ denote the number of customers in the orbit at the time t. If at the time t the server is busy and service rate is equal to μ_i, $i \geqslant 1$, then we say that the server is in the phase i. If at the time t the server is free we say that the server is in the phase 0. Let $\eta(t) \in \{0, 1, 2, \ldots, m+1\}$ denote the phase of the server at time t. The process $(\eta(t), \xi(t))$ is a homogeneous Markov process with state space $E = \{(i,j) : 0 \leqslant j \leqslant m, 0 \leqslant i \leqslant j+1\}$.

Theorem. *The ergodic distribution of the process $(\eta(t), \xi(t))$ can be presented as follows*

$$
\begin{aligned}
\pi_{ij} &= \pi_{00} \frac{\beta_i^{j-i+1} A_i}{(i-1)!\mu_i}, && 0 \leqslant i-1 \leqslant j \leqslant m-1,\\
\pi_{0j} &= \pi_{00} \frac{\lambda}{j\nu} \sum_{k=1}^{j} \frac{\beta_k^{j-k} A_k}{(k-1)!\mu_k}, && 1 \leqslant j \leqslant m,\\
\pi_{im} &= \pi_{00} \frac{\lambda \beta_i^{m-i} A_i}{(i-1)!\mu_i^2}, && 1 \leqslant i \leqslant m,\\
\pi_{00} &= \Big(1 + \sum_{k=1}^{m} d(k,m) A_k\Big)^{-1}, \ \pi_{m+1m} = \frac{\pi_{00}\lambda^2}{m\nu\mu_{m+1}} \sum_{k=1}^{m} \frac{\beta_k^{m-k} A_k}{(k-1)!\mu_k},
\end{aligned}
$$

where

$$
\beta_i = \frac{\lambda}{\lambda + \mu_i}, \quad d(k,m) = \frac{1}{(k-1)!\mu_k} \left[\frac{\lambda + \mu_k}{\mu_k} + \frac{\lambda^2 \beta_k^{m-k}}{m\nu\mu_{m+1}} + \frac{\lambda}{\nu} \sum_{l=0}^{m-k} \frac{\beta_k^l}{l+k} \right],
$$

and the function $H(i,k)$ is defined in the process of proof.

Some optimization problems for such systems are considered.

References

[1] G. I. Falin, J. G. C. Templeton, *Retrial Queues*, Chapman and Hall, London, 1997.