MSc. Grzegorz Sługocki
Warsaw University of Technology, Faculty of Power and Aerospace Engineering
DSc. Michał Wychowański
AWF Warsaw, Faculty of Rehabilitation
Ph.D. Grzegorz Orzechowski
Laapenranta University of Technology, Dept. of Mechanical Engineering
Ph.D. Dariusz Radomski
Warsaw University of Technology, Institute of Radioelectronics and Multimedia Techniques

The hydrodynamic flow past the oar and mathematical description of the oar force

A flow past the cyclically moving oar is a unsteady 3D phenomenon which is described by Navier–Stokes Equations for incompressible, viscous and not heat conductive fluid. This phenomenon emphasizes the flow with wake behind the oar blade which is considered as an airfoil of small aspect ratio [1]. The Navier–Stokes equations are:

$$\operatorname{div}(\mathbf{V}) = 0 \tag{1}$$

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} = \mathbf{F} - \frac{1}{\rho}\operatorname{grad} p + \nu\Delta\mathbf{V} \tag{2}$$

$$\frac{d}{dt}\Big(\frac{p}{\rho} + 0.5V^2\Big) = \mathbf{F}\cdot\mathbf{V} - \frac{1}{\rho}\operatorname{div}(p\mathbf{V}) + \nu(\Delta\mathbf{V})\cdot\mathbf{V} - \frac{1}{\rho}\frac{\partial p}{\partial t} \tag{3}$$

The boundary conditions are:

$$\mathbf{V}_\infty(t) = -\dot{\gamma} l_{OA} \quad p = p_\infty \tag{4}$$

$$\mathbf{V}_{OA}\cdot\mathbf{n}_{OA} = 0 \quad \mathbf{V}_{OA} - (\mathbf{V}_{OA}\cdot\mathbf{n}_{OA})\mathbf{V}_{OA} = 0 \tag{5}$$

The solution of the above shown system gives the velocity $\mathbf{V}$ field and the pressure p in the volumic domain Ω external to oar blade. The hydrodynamic force and torque exerting on the blade are as follows:

$$\mathbf{P} = -\iint\limits_{\partial\Omega_{OA}} p_{OA}(\gamma)\mathbf{n}_{OA} d(\partial\Omega), \quad \mathbf{M} = -\iint\limits_{\partial\Omega_{OA}} p_{OA}(\gamma)(\mathbf{r}_{OA}\times\mathbf{n}_{OA}) d(\partial\Omega) \tag{6}$$

The lift and drag forces acting on the oar blade as well as the both normal and tangent forces to the oar blade chord are as follows:

$$\mathbf{P}_L = \mathbf{P}\cos\alpha = 0.5\rho c_L(\gamma)\int\limits_{l_{OAmin}}^{l_{OAmax}} [l_{OA}\dot{\gamma}(t)]^2 b_{OA}(l_{OA})\, dl_{OA}$$

$$\alpha = 0.5\pi - \arccos\Big(\frac{\mathbf{V}_\infty\cdot\mathbf{P}}{|\mathbf{V}_\infty||\mathbf{P}|}\Big) \tag{7}$$

$$\mathbf{P}_D = \mathbf{P}\sin\alpha = 0.5\rho c_D(\gamma) \int\limits_{l_{OAmin}}^{l_{OAmax}} [l_{OA}\dot{\gamma}(t)]^2 b_{OA}(l_{OA})\, dl_{OA}$$

$$\alpha = 0.5\pi - \arccos\Big(\frac{\mathbf{V}_\infty \cdot \mathbf{P}}{|\mathbf{V}_\infty||\mathbf{P}|}\Big) \tag{8}$$

$$\mathbf{P}_N = \mathbf{P}_L\cos(\gamma) + \mathbf{P}_D\sin(\gamma), \quad \mathbf{P}_T = -\mathbf{P}_L\sin(\gamma) + \mathbf{P}_D\cos(\gamma). \tag{9}$$

Then the hydrodynamic lift c_L and drag c_D coefficients vs. oar geometrical incidence angle γ are:

$$c_L(\gamma) = \frac{2\mathbf{P}_L}{\rho \int\limits_{l_{OAmin}}^{l_{OAmax}} [l_{OA}\dot{\gamma}(t)]^2 b_{OA}(l_{OA})\, dl_{OA}}$$

$$c_D(\gamma) = \frac{2\mathbf{P}_D}{\rho \int\limits_{l_{OAmin}}^{l_{OAmax}} [l_{OA}\dot{\gamma}(t)]^2 b_{OA}(l_{OA})\, dl_{OA}} \tag{10}$$

For the simplest description of such phenomenon it is allowed to take the value $c_D(0.5\pi)$ and c_L=0 for the range of geometrical incidence angle γ containing the value 0.5π. This means that it is enough to take only the perpendicular force component to the blade surface rotating around a fixed point, distant of l_{OA} from the origin of cartesian coordinates fixed to the oar blade.

References

[1] L. M. Milne-Thomson. *Theoretical Hydrodynamics*. MacMillan, London, 1970.

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[3] W. Bollay. *A Non-Linear Wing Theory and its Application to Rectangular Wings of Small Aspect Ratio*. ZAMM 19 (1939), No. 1.

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