



EUROPEAN COMMISSION
RESEARCH DG HUMAN RESOURCES
AND MOBILITY

TOK Periodic Activity Report

Project No.: 14508

Project Acronym: SPADE2

Project Full Name: DETERMINISTIC AND STOCHASTIC
DYNAMICS, FRACTALS, TURBULENCE

Marie Curie Actions

TOK Periodic Activity Report

Period covered: from 01/09/2006 to 31/08/2007

Date of preparation: 12/10/2007

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Project coordinator name:
Prof Feliks Przytycki

Duration: 48

Project coordinator organisation name:
Institute of Mathematics of the Polish Academy of
Sciences

Version: 1

Marie Curie Actions

TOK Periodic Activity Report

GENERAL INFORMATION

| | |
|---|---|
| Project No.: | 14508 |
| Project acronym: | SPADE2 |
| Project full name: | DETERMINISTIC AND STOCHASTIC DYNAMICS, FRACTALS, TURBULENCE |
| Period number: | 2nd |
| Period covered - start date: | 01/09/2006 |
| Period covered - end date: | 31/08/2007 |
| Project start date: | 01/09/2005 |
| Project duration [months]: | 48 |
| Project coordinator name: | Prof Feliks Przytycki |
| Project coordinator organisation name: | Institute of Mathematics of the Polish Academy of Sciences |
| Date of submission: | 25/09/2007 |

SUMMARY OF THE RECRUITMENT DURING THE REPORTING PERIOD

Contractor: Institute of Mathematics of the Polish Academy of Sciences

| Name of the Researcher (as stated at time of selection) | Experience | Type | Origin | | Gender | Start date of recruitment | End date of recruitment | Working time commitment | No. of full-time equivalent months covered by this recruitment during the reporting period |
|---|------------------|----------|-----------------------|-----|--------|---------------------------|-------------------------|-------------------------|--|
| | | | Country | LFR | | | | | |
| Michail Zhitomirskii | MER (> 10 years) | Incoming | IL-Israel | No | Male | 01/09/2006 | 31/10/2006 | Full Time | 2.0 |
| Maxim Arnold | ER (4-10 years) | Incoming | RU-Russian Federation | No | Male | 01/10/2006 | 31/01/2007 | 4.0 | Full Time |
| Andrzej T. Szankowski | MER (> 10 years) | Incoming | IL-Israel | No | Male | 01/04/2007 | 31/05/2007 | 2.0 | Full Time |
| Smbat Gogyan | ER (4-10 years) | Incoming | AM-Armenia | No | Male | 18/09/2006 | 17/09/2007 | 12.0 | Full Time |
| Katrin Gelfert | ER (4-10 years) | Incoming | DE-Germany | No | Female | 12/02/2007 | 11/05/2007 | 3.0 | Full Time |

Contractor: The University of Warwick

| Name of the Researcher (as stated at time of selection) | Experience | Type | Origin | | Gender | Start date of recruitment | End date of recruitment | Working time commitment | No. of full-time equivalent months covered by this recruitment during the reporting period |
|---|-----------------|----------|-----------|-----|--------|---------------------------|-------------------------|-------------------------|--|
| | | | Country | LFR | | | | | |
| Witold Sadowski | ER (4-10 years) | Outgoing | PL-Poland | Yes | Male | 01/10/2006 | 31/01/2007 | Full Time | 4.0 |

Contractor: Universite Pierre et Marie Curie - Paris 6

| Name of the Researcher (as stated at time of selection) | Experience | Type | Origin | | Gender | Start date of recruitment | End date of recruitment | Working time commitment | No. of full-time equivalent months covered by this recruitment during the reporting period |
|---|------------------|----------|-----------|-----|--------|---------------------------|-------------------------|-------------------------|--|
| | | | Country | LFR | | | | | |
| Przemyslaw Wojtaszczyk | MER (> 10 years) | Outgoing | PL-Poland | Yes | Male | 03/03/2007 | 02/06/2007 | Full Time | 3.0 |

Contractor: Scuola Normale Superiore di Pisa

| Name of the Researcher (as stated at time of selection) | Experience | Type | Origin | | Gender | Start date of recruitment | End date of recruitment | Working time commitment | No. of full-time equivalent months covered by this recruitment during the reporting period |
|---|------------|------|---------|-----|--------|---------------------------|-------------------------|-------------------------|--|
| | | | Country | LFR | | | | | |

Contractor: L'Institut National de Recherche en Informatique et en Automatique

| Name of the Researcher (as stated at time of selection) | Experience | Type | Origin | | Gender | Start date of recruitment | End date of recruitment | Working time commitment | No. of full-time equivalent months covered by this recruitment during the reporting period |
|---|------------------|----------|-----------|-----|--------|---------------------------|-------------------------|-------------------------|--|
| | | | Country | LFR | | | | | |
| Michal Rams | MER (> 10 years) | Outgoing | PL-Poland | Yes | Male | 01/01/2007 | 28/02/2007 | Full Time | 2.0 |

Contractor: Christian-Albrechts Unversitaet zu Kiel

| Name of the Researcher (as stated at time of selection) | Experience | Type | Origin | | Gender | Start date of recruitment | End date of recruitment | Working time commitment | No. of full-time equivalent months covered by this recruitment during the reporting period |
|---|------------|------|---------|-----|--------|---------------------------|-------------------------|-------------------------|--|
| | | | Country | LFR | | | | | |

GENERAL PROGRESS OF THE PROJECT

Please indicate if the project

a) is, at this stage, being implemented as originally planned

If you answered b) or c) please include a detailed description of the modifications in the report

Qualitative indicators of progress and success

The programme has been organized in 4 Tasks. A description of the progress in the second period.

1. Dynamical Systems. There is a further progress in understanding conformal and invariant measures for iteration of entire and meromorphic functions, by M. Urbanski, A. Zdunik, J. Kotus, K. Baranski, B. Karpinska, and for rational functions. A paper on uniqueness and analyticity of pressure for TCE (non-uniformly hyperbolic) rational maps by F. Przytycki, J. Rivera-Letelier and S. Smirnov is in preparation. M. Rams visited INRIA for the second time. A remarkable paper was written joint with J. Levy-Vehel concerning Traffic Control Protocol (TCP), using methods of multifractal analysis. On "incoming visit" Katrin Gelfert studied with M. Rams real 1D nonuniformly hyperbolic IFS.

1a. Modelling processes in ecology. A number of results have been obtained in relation with Warsaw and Katowice branch seminars. Rudnicki with collaborators in several papers have analyzed a probabilistic model of genome evolution and asymptotics (in relation with Warsaw bioinformatics seminar at IMPAN), random walks on neural networks, and also models for animals aggregations.

2. PDE's, turbulence, asymptotics. On the "incoming visit" by Maxim Arnold, former PhD student of Ya. Sinai, solutions of Navier-Stokes were discussed with members of Warsaw group: W. Zajaczkowski, P. Mucha, J. Renclawowicz, at Warsaw University and IMPAN seminars on PDE's and Fluid Dynamics, with results in understanding 3D NS, in special domains, and 2D. The motions considered in their work describe blood motions in vessels. G. Lukaszewicz and W. Sadowski (on "outgoing visit" to Warwick partner, cooperating with James C. Robinson) worked on attractors; numerical methods in 3D have been developed.

3. Stochastic Processes, scaling limits. T. Komorowski, Sz. Peszat and T. Szarek are preparing a paper on invariant measures for stochastic PDE's. Hilbert space valued Levy processes have also been studied. L. Stettner with collaborators worked in stochastic processes and financial mathematics. Remarkable are the papers "Ergodicity of filtering process by vanishing discount approach" by G. B. Di Massi, and "Heavy traffic analysis for EDF queues with reneging" by L. Kruk et al.. Two regular seminars have been working at IMPAN: "Financial Mathematics" and "Stochastik Processes". There were no long term visits in the reported period.

3a. Control, subriemannian geometry, singularities. On an "incoming visit" within SPADE by M. Zhitomirsky remarkable papers with S. Janeczko and W. Domitrz have been written, concerning symplectic singularities of varieties (one will appear in *J. für die Reine und Angewandte Mathematik*). Caustics conference was organized.

4. Function Spaces. Smbat Gogyan (from Armenia) spent 1 year on SPADE2 junior researcher position in Warsaw and Sopot branch, working with P. Wojtaszczyk, Anna Kamont and Z. Ciesielski in approximation theory (greedy bases), also with P.F.X. Mueller (a shorter visit). Wojtaszczyk visited ("outgoing") Paris 6, learning numerical (wavelets) methods (A. Cohen), and in Equipe d'Analyse Fonct. and Lab. J.-L. Lions. Related were studies by A. T. Szankowski (Hebrew U., Jerusalem) "incoming" with T. Figiel and A. Pelczynski on bases in function spaces. Cooperation with Koldobsky ("incoming" in the previous period) continued in geometry of Banach spaces. M. Wojciechowski continued his collaboration with Giovanni Alberti in approximation theory.

The progress in all the tasks is satisfactory. IMPAN has become an international mathematical center, successful in transfer of knowledge, in cooperation with Warsaw University and Warsaw University of Technology.

PROJECT ACHIEVEMENTS

Scientific highlights

1. Dynamical Systems

Katrin Gelfert (incoming) studied non-uniformly hyperbolic dynamical systems using thermodynamical formalism, in particular pressure via periodic orbits in [GW, G] K. Gelfert and M. Rams also studied a class of iterated function systems of the real line as one of the most basic model problems in the non-hyperbolic setting. In [GR1, GR2] they gave a complete analysis of the geometric and dynamical properties as well as of the dimension spectrum of the Lyapunov exponents. This included investigations of expansive Markov system without a priori requiring any uniformity.

J. Levy-Vehel and Michal Rams (visit to INRIA) studied data transmission processes (Traffic Control Protocol - TCP) by methods of multifractal analysis, [L-VR]. The processes bear some analogy with Levy processes, but their increments are neither stationary nor independent.

1a. Modelling processes in ecology.

In [Ru1] the authors introduce and analyze a simple probabilistic model of genome evolution. It is based on three fundamental evolutionary events: gene loss, duplication and accumulated change. The authors are mainly interested in asymptotic size distribution of small paralogous gene families in a genome. This is motivated by previous works which consisted in fitting the available genomic data into, what is called, paralog distributions. This formalism is described as a discrete-time Markov chain. The formulas for equilibrium paralog family sizes are derived. Some empirical results for microbial genomes are presented.

In [Ru2] a directed-edge-reinforced random walk on graphs is considered. Criteria for the walk to end up in a limit cycle are given. Asymptotic stability of some neural networks is shown. The purpose of [Ru3] is to propose a two-dimensional model describing the dynamics of animal aggregation and to investigate it analytically. The movement of animals is described by a PDE. The authors analyze the properties of the solutions of this equation by means of the theory of Markov semigroups and prove the existence of a stationary solution.

2. Partial Differential Equations.

Several results concern solutions of Navier-Stokes Eq. in special domains. continuing investigations from the first period. J. Renclawowicz and W. Zajaczkowski proved in [R-Z] the large time existence of solutions to the Navier-Stokes equations with slip boundary conditions in a cylindrical domain. The motions considered in the paper could describe blood motions in vessels. K. Pileckas in [Pi] studied three-dimensional nonstationary Navier-Stokes problem in unbounded domains with cylindrical outlets to infinity. A general regularity and stability theory has been developed by P. Mucha. In [Mu1] the author examines states of Stefan-type problems in the plane with the Gibbs-Thomson correction involving general anisotropic energy density function. In [Mu2] the author proves existence of global in time regular solutions to the NS equ. in an arbitral three dimensional domain with a general boundary condition. In [Mu3] the author investigates the global in time stability of regular solutions with large velocity vectors to the evolutionary N-S equ. in R^3 . M. Arnold jointly with P. Mucha discussed a new method to deal with the solutions of 2d N-S systems, having measures as initial data.

Gevrey regularity in time variable of genuine and formal solutions to the initial value problem for semilinear heat equation was studied by G. Lysik and T. Gramchev in [G-L] and by G. Lysik and S. Michalik in [L-M].

In the mainstream of SPADE2 are papers by G. Lukaszewicz and W. Sadowski on attractors for N-S Equ. In [Lu2] the author considers a two-dimensional Navier-Stokes shear flow with time dependent boundary driving and subject to Tresca law. In [Lu1] the author investigates some relations among the notions of pullback attractor, time-average measure and statistical solution. W. Sadowski considers behaviour of the weak solutions of the unforced N-S 3D equations with periodic boundary conditions. In the joint paper with J.C. Robinson [S-R] authors present a method of numerical verification whether a given bounded set of initial conditions gives rise to regular solutions of the 3D N-S equations.

The results by T. Reginska concern numerical methods. [R-W] concerns a problem of reconstruction of acoustic or electromagnetic field from unexact data given on an open part of a boundary of a given domain.

3. Stochastic Processes, scaling limits.

In [MStet] by Di Messi and L. Stettner a new original approach of the study of ergodicity of filtering processes is proposed. Ergodicity of filtering processes is a one of most challenging problems in the systems theory. It is expected that this paper may open a new direction of research. L. Kruk in [Kruk] presents a heavy-traffic analysis of the behavior of a single-server queue under an Earliest-Deadline-First (EDF) scheduling policy. The performance of the system is measured by the fraction of reneged work (the residual work lost due to elapsed deadlines), which is shown to be minimized by the EDF policy. The evolution of the lead time distribution of customers in queue is described by a measure-valued process. The heavy traffic limit of this (properly scaled) process is shown to be a deterministic function of the limit of the scaled workload process, which, in turn, is identified to be a doubly reflected Brownian motion.

3a. Control, subriemannian geometry, singularities.

In [DJZ] by S. Janeczko, W. Domitrz and M. Zhitomirsky the problem of local symplectic classification of varieties was considered. It was realized that "ghost" invariants can be explained in simple terms. This was the starting point for developing the method of algebraic restrictions of differential forms to singular varieties. It occurred to be a powerful tool for the symplectic classification of a wide class of varieties.

4. Function spaces.

A. Kalamajska and K. Pietruska-Paluba continued a study of inequalities between Orlicz norms in function spaces in [K-P].

In [Ko1] the solution the famous slicing problem is given for certain classes of bodies using a generalization of Khinchin's inequality. In [Ko2] a complex version of the Busemann-Petty problem is solved. The problem asks whether symmetric convex bodies with smaller areas of central hyperplane sections necessarily have smaller volume.

Smbat Gogyan worked with P. Wojtaszczyk and prepared paper [Gogyan] on quasi-greedy subsystems in Haar systems. He also worked with Anna Kamont and Z. Ciesielski and studied the Feymann-Kac formula.

Paul F.X. Mueller and Anna Kamont (IMPAN) investigated a problem by E.M. Semenov, about Semenov's characterization of rearrangements of the Haar system.

A. T. Szankowski (1 year visit to IMPAN, 2 months on SPADE2), in collaboration with T. Figiel developed the framework for investigation the approximation property in the space of bounded analytic functions on the unit disc; and together with A. Pelczynski gave the estimate on the order of magnitude of the unconditional basis constant of spaces H_p .

M. Wojciechowski following his study initiated during his visit in Pisa, proved jointly with M. Roginskaya bounded approximation property of homogeneous Sobolev spaces on arbitrary simply connected planar domains.

Teaching and training activities

Seminars, workshops and conferences organized by/at IMPAN were lectures by in relation with SPADE2 were given:

* IMPAN and Warsaw University "Dynamical Systems" seminars: M. Rams, K. Gelfert, J. Levy-Vehel, E. Gutkin, C. Cabrera, M. Arnold, G. Levin.

* "Spring Dynamical Systems School" in Bedlewo, April 2007, 40 young participants, org: M. Rams, K. Fraczek. Some lectures (courses): E. Gutkin "Introduction to billiards", Tomasz Szarek "Invariant measures for Markov processes", Marta Tyran-Kaminska "Central limit theorems in dynamical systems", P. Walczak "Denjoy theorems in foliations"

* "Algebraic Actions of Higher Rank Abelian Groups and Introduction to Rigidity" Lectures of Anatole Katok, 26 February - 9 March, IMPAN, around 20 young participants from Poland, 3 from abroad.

* IMPAN and Warsaw University Seminars "PDE's and Fluid Dynamics" and "Navier-Stokes Equations and PDE's": P. Mucha, W. Sadowski, G. Lukaszewicz, W. Zajackowski, M. Arnold, J. Renclawowicz.

* Polish-Corean Workshop, IMPAN, May 6-12, around 25 participants. There were interesting lectures of H.Choe (Regularity problem for the Navier-Stokes equations) and Bae (Existence and regularity of solutions to elliptic system). From the polish side we had lectures of P.Mucha (Uniqueness for

Euler equations and Zygmund spaces), J. Renclawowicz (Regularity to Navier-Stokes equations in a cylinder), P. Gwiazda, E. Zadrzyska (Parabolic equations in Besov spaces), I. Pawlow (Cahn-Hilliard system in an elastic solids), W. Zajaczkowski (Regularity to the Navier-Stokes equations) and some young scientists (students and PhD students).

- * Seminar "Analytic Theory of Differential Equations" at IM PAN: G. Lysik, S. Michalik.
- * Seminars at IMPAN on "Financial Mathematics and Stochastic Processes" about 20 participants
- * "Advances in Mathematics of Finance", Conference, Bedlewo, Apr. 30 -- May 5. Plenary speakers: Freddy Delbaen (Zurich), Ernst Eberlein (U. Freiburg), Yuri Kabanov (U. Besancon), Piotr Karasinski (HSBC London), Alex Lipton (Merrill Lynch), Marek Musiela (Paribas), Bernt Oksendal (Oslo).
- * Advanced Mathematical Methods for Finance (AMaMeF), a conference by European Science Foundation Network organized by Lukasz Stettner.
- * "Seminar on Singularities" at Warsaw Technological University: Zhitomirsky, Janeczko, Domitrz.
- * "Geometric Singularity Theory", Polish-Japanese Workshop, IMPAN Sopot branch, Aug 2007.
- * A session devoted to zeta functions, IMPAN, 4-5 May,
- * "Seminar in Functional Analysis" at IMPAN.
- * "Geometric Analysis and Nonlinear Partial Differential Equations" - European Mathematical Society Summer School and Conference: B. Bojarski, P. Goldstein, P. Hajlasz, T. Iwaniec, P. Strzelecki.
- * "Seminar for PhD students", lecturers included: S. Gogyan, A. Szankowski, M. Rams, J. Renclawowicz, A. Katok.
- * Sessions in all the tasks were organized also during the American Mathematical Society -- Polish Mathematical Society joint meeting, Warsaw, July/August 2007.

Dissemination of results

Publications written within the SPADE 2 program (some were already listed in Report I as preprints)

- [GW] K. Gelfert and Ch. Wolf "Topological pressure for one-dimensional holomorphic dynamical systems" Bull. Polish Acad. Sci. Math. 55 (2007), 53--62.
- [G] K. Gelfert "Equality of pressures for diffeomorphisms preserving hyperbolic measures", subm..
- [GR1] K. Gelfert and M. Rams, "Geometry of limit sets for expansive Markov systems", subm.
- [GR2] K. Gelfert and M. Rams, "The Lyapunov spectrum of some parabolic systems", subm.
- [L-VR] J. Levy-Vehel, M. Rams "Large Deviation Multifractal Analysis of a Class of Additive Processes with Correlated Non-Stationary Increments"
- [Ru1] R. Rudnicki and J. Tiuryn and D. Wojtowicz, "A discrete model of evolution of small paralog families", Math. Models and Methods in Appl. Sci., 17 (2007), 933--955.
- [Ru2] R. Rudnicki and J. Myjak, "Reinforced walk on graphs and neural networks", subm..
- [Ru3] R. Rudnicki and M. Adioui, "A two-dimensional model of animal grouping", subm..
- [R-Z] J. Renclawowicz, W. Zajaczkowski "Large time regular solutions to the Navier Stokes equations in cylindrical domains", to appear in TMNA
- [Arnold] M. Arnold "Analyticity of Periodic Solutions of the 2d Boussinesq System", Intern. Math. Ser., 6, Instability in Models Connected with Fluids Flow, 2007-2008.
- [G-L] T. Gramchev, G. Lysik, "Uniform analytic-Gevrey regularity of solutions to a semilinear heat equation", to appear in Banach Center Publ.
- [L-M] G. Lysik, S. Michalik, "Formal solutions of semilinear heat equations", to appear in J. Math. Anal. Appl..
- [Pi] K. Pileckas, "Solvability in weighted spaces of the three-dimensional Navier-Stokes problem in domains with cylindrical outlets to infinity", Topological Methods in Nonlinear Analysis, 2007, 27p.
- [Lu1] G. Lukaszewicz, "Pullback attractors and statistical solutions of 2D Navier-Stokes equations", to appear Discrete and Continuous Dynamical Systems.
- [Lu2] G. Lukaszewicz "Pullback attractors for 2D Navier-Stokes equations with Tresca boundary condition", to appear in Banach Center Publ.
- [Mu1] P.B. Mucha, P. Rybka, "A new look on equilibrium for Stefan type problems in the plane", SIAM Journal on Mathematical Analysis, accepted.
- [Mu2] P.B. Mucha, "Global solutions, structure of initial data and the Navier-Stokes equations", preprint.
- [Mu3] P.B. Mucha, "Stability of 2D incompressible flows in R^3 ", preprint.
- [S-R] W. Sadowski, J. C. Robinson "Decay of weak solutions and the singular set of the three

dimensional Navier-Stokes equations", *Nonlinearity*, 20 (2007), 1185-1191;

[R-W] T. Reginska, A. Wakulicz "Wavelet moment method for Cauchy problem for the Helmholtz equation"

[MStet] G.B. Di Massi, L. Stettner, "Ergodicity of filtering process by vanishing discount approach", to appear in *Systems and Control Letters*,

[Kruk] L. Kruk, Lehoczky, Ramanan, Shreve "Heavy traffic analysis for EDF queues with reneging" in preparation.

[Kuc] L. Kucinski, "Equivalence of measures corresponding to the Hilbert space valued Levy processes" in preparation.

[DJZ] W.Domitrz, S.Janeczko, M.Zhitomirskii, "Symplectic singularities of varieties: the method of algebraic restrictions", to appear in *J. Reine Angew. Anal.*

[K-P] Agnieszka Kalamajska, Katarzyna Pietruska-Paluba, "Gagliardo-Nirenberg inequalities in weighted Orlicz spaces equipped with a not necessarily doubling measure", to appear in *Bull. Belgian Math. Soc.*

[Ko1] A.Koldobsky, A.Pajor, V.Yaskin, "Inequalities of the Khinchin type and sections of L_p -balls", *Studia Math.* subm.

[Ko2] A.Koldobsky, H.Konig, M.Zymonopoulou, "The complex Busemann-Petty problem on sections of convex bodies", *Math. Annalen*, subm.

[Gogyan] Smbat Gogyan "On the convergence of greedy algorithm in $L_1(0,1)$ ", subm.

[Szan] A. T. Szankowski "Three space problems for the approximations property", preprint.

[Woj] M. Wojciechowski "Bounded Approximation Property of Sobolev Spaces on Arbitrary Planar Domains", preprint.

In Annex 1 see lectures on conferences, workshops and university seminars.

ADDITIONAL INFORMATION

Please indicate any additional information, which may be considered useful to assess the work done during the reporting period. The socio-economic aspect of the project may be addressed in this section.

The Polish Ministry of Science and Higher education awarded the project a matching grant of about 450 000 euros. This grant is used to support short visits at IMPAN of foreign and Polish (from other cities) mathematicians and also short outgoing visits, substantial for the success of the program, supports schools and workshops, enables us also to buy some necessary computers, programs etc.

SPADE2 is cooperating with other FP6 Marie Curie programmes at IMPAN, in particular RTN "Conformal Structures and Dynamics" acr. CODY, that have started in January 2007, is coordinated at Warwick, and has a node at IMPAN.

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|--------------------|-------------|
| Attachments | Annex I.doc |
| Name | |
| Date | |
| Signature | |