



EUROPEAN COMMISSION
RESEARCH DG HUMAN RESOURCES
AND MOBILITY

TOK Periodic Activity Report

Project No.: 14508

Project Acronym: SPADE2

Project Full Name: DETERMINISTIC AND STOCHASTIC
DYNAMICS, FRACTALS, TURBULENCE

Marie Curie Actions

TOK Periodic Activity Report

Period covered: from 01/09/2005 to 31/08/2006

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Project coordinator name:
Prof Feliks Przytycki

Duration: 48

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Institute of Mathematics of the Polish Academy of
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Marie Curie Actions

TOK Periodic Activity Report

GENERAL INFORMATION

Project No.:	14508
Project acronym:	SPADE2
Project full name:	DETERMINISTIC AND STOCHASTIC DYNAMICS, FRACTALS, TURBULENCE
Period number:	1st
Period covered - start date:	01/09/2005
Period covered - end date:	31/08/2006
Project start date:	01/09/2005
Project duration [months]:	48
Project coordinator name:	Prof Feliks Przytycki
Project coordinator organisation name:	Institute of Mathematics of the Polish Academy of Sciences
Date of submission:	12/10/2006

SUMMARY OF THE RECRUITMENT DURING THE REPORTING PERIOD

Contractor: Institute of Mathematics of the Polish Academy of Sciences

Name of the Researcher (as stated at time of selection)	Experience	Type	Origin		Gender	Start date of recruitment	End date of recruitment	No. of full-time equivalent months covered by this recruitment during the reporting period
			Country	LFR				
Konstantin Pileckas	ER (> 10 years)	Incoming	LT-Lithuania	Yes	Male	23/01/2006	23/04/2006	3
Michel Zinsmeister	ER (> 10 years)	Incoming	FR-France	No	Male	27/02/2006	26/04/2006	2
Alekkxander Koldobskiy	ER (> 10 years)	Incoming	US-United States	No	Male	13/03/2006	12/06/2006	3

Contractor: The University of Warwick

Name of the Researcher (as stated at time of selection)	Experience	Type	Origin		Gender	Start date of recruitment	End date of recruitment	No. of full-time equivalent months covered by this recruitment during the reporting period
			Country	LFR				

Contractor: Universite Pierre et Marie Curie - Paris 6

Name of the Researcher (as stated at time of selection)	Experience	Type	Origin		Gender	Start date of recruitment	End date of recruitment	No. of full-time equivalent months covered by this recruitment during the reporting period
			Country	LFR				

Contractor: Scuola Normale Superiore di Pisa

Name of the Researcher (as stated at time of selection)	Experience	Type	Origin		Gender	Start date of recruitment	End date of recruitment	No. of full-time equivalent months covered by this recruitment during the reporting period
			Country	LFR				
Michal Wojciechowski	ER (> 10 years)	Outgoing	PL-Poland	Yes	Male	01/03/2006	31/05/2006	3

Contractor: L'Institut National de Recherche en Informatique et en Automatique

Name of the Researcher (as stated at time of selection)	Experience	Type	Origin		Gender	Start date of recruitment	End date of recruitment	No. of full-time equivalent months covered by this recruitment during the reporting period
			Country	LFR				
Michal Rams	ER (4-10 years)	Outgoing	PL-Poland	Yes	Male	01/01/2006	28/02/2006	2

Contractor: Christian-Albrechts Unversitaet zu Kiel

Name of the Researcher (as stated at time of selection)	Experience	Type	Origin		Gender	Start date of recruitment	End date of recruitment	No. of full-time equivalent months covered by this recruitment during the reporting period
			Country	LFR				
Teresa Reginska	ER (> 10 years)	Outgoing	PL-Poland	Yes	Female	01/11/2005	31/12/2005	2

GENERAL PROGRESS OF THE PROJECT

Please indicate if the project

a) is, at this stage, being implemented as originally planned

If you answered b) or c) please include a detailed description of the modifications in the report

Qualitative indicators of progress and success

The programme has been organized in 4 Tasks.

1. Dynamical Systems.

There is a progress in understanding conformal and invariant measure for iteration of holomorphic and meromorphic maps (Przytycki, Kotus, Zdunik, Urbanski). In the study of dependence of Hausdorff dimension of Julia sets on parameter (L. Jaksztas) consultations by "incoming" M. Zinsmeister were substantial. M. Rams on "outgoing" visit to INRIA Rocquencourt (Partner 5) studied with J. Levy-Vehel behaviour of dimension spectra of self-similar fractals under projections. They wrote a joint paper. Two weekly seminars in Warsaw took place, with many talks by visitors, and two small workshops/schools: One in 1D dynamics and one in Fractals. Several top mathematicians took place in them (P. Mattila, Ch. Bandt, J. Levy-Vehel, M. Urbanski, H. Bruin, M. Denker). A 3-days Spring school was organized in Dynamical Systems with 40 young participants from Poland.

1a. On the border of Dynamical Systems, PDE's and Stochastic Processes were papers by R. Rudnicki and collaborators on modelling processes in ecology. An "incoming" visit in this area will take place in the second year. This group activity inspired recently creation of national mathematics+physics+biology network CeZaM.

2. PDE's, turbulence, asymptotics.

Fruitful was the "incoming" visit by Konstantin Pileckas. He gave several talks and a mini-course at Warsaw PDE's seminars. Several papers by him, W. Zajackowski and J. Renclawowicz on existence of solutions of Navier-Stokes Equation were written. Some concern flows in pipes, modelling blood flow. A conference "Parabolic and Navier-Stokes Equations" was attended by many specialists from Poland, Germany, Russia, Japan, Portugal, Czech and other countries. T. Reginska on "outgoing" visit to Kiel (Partner 6) worked on modelling the propagation of radiation field in solids. Some results (P. Mucha, G. Lukaszewicz) concern stability and attractors of N-S Eq.; we plan more activities in this next years (due to several "outgoing" visits planned soon).

3. Stochastic Processes, scaling limits.

On "incoming visit" M. Zinsmeister had a 24 hours course (and exercises for students) on SLE, Stochastic (Schramm) Loewner Evolution (notes available on web page: <http://www.univ-orleans.fr/mapmo/membres/zins/SLE/>) with almost 20 participants (including several PhD students). (This new theory was created in recent years by Lawler, Schramm and Werner, the latter got in Madrid ICM in August 2006 Fields medal.) He had several talks, in particular on random growth processes. Within SPADE2 J. Zabczyk worked with Z. Priola (a joint paper has been accepted for publication). Also A. Rusinek prepared a paper on stochastic evolution equations.

4. Function Spaces.

Fruitful was "incoming" visit by A. Koldobsky. He gave several talks at Functional Analysis and at Probability seminars at IMPAN, and Warsaw and Wroclaw Universities on geometric functional and Fourier analysis (some related to tomography) and prepared two publications. M. Wojciechowski at the "outgoing" visit to Pisa (Partner 4), working with G. Alberti, obtained results concerning the interplay between geometric measure theory and geometry of function spaces, gaining some planned objectives in approximation theory. A. Kalamajska et al. obtained several results in analysis on metric spaces.

A multidisciplinary SPADE2 monthly seminar was organized, several other courses in the topics of

SPADE2 took place at IMPAN and Warsaw University, with participation of SPADE2 visitors. There were 4 meetings of SPADE2 steering committee and its selection subcommittee. SPADE2 web page www.impan.gov.pl/EU/ has been created. Contacts with partners were quite large, especially with Pisa. The progress in all the Task (research and training) is satisfactory, some topics more advanced some less.

PROJECT ACHIEVEMENTS

Scientific highlights

1. Dynamical Systems

a) M. Rams on his visit to the SPADE2 partner INRIA Rocquencourt wrote with J. Levy-Vehel the paper "Results on the Dimension Spectrum for Self-Conformal Measures"

<http://www.impan.gov.pl/~rams> . Dimension spectrum D_1 describes geometrical structure of measures in \mathbb{R}^n . It is a function, giving for different values of parameter l value of a corresponding dimension of the measure. In particular, $l=0$ correspond to the Minkowski (box) dimension, $l=2$ to the correlation dimension and $l=1$ to the entropy dimension. The authors contributed to the theory started by Hunt, Kaloshin, and Jarvenpaa's) on behaviour of D_1 under transversal projections for limits of Iterated Function Systems.

b) F. Przytycki, J. Kotus, A. Zdunik and co-authors made a progress in understanding conformal measures in iteration of rational and meromorphic functions.

c) A highlight are papers by R. Rudnicki and co-authors that are applications of methods of PDE's, Dynamical Systems and Stochastic Processes to modelling of processes in ecology. These are

[RW1] R. Rudnicki and R. Wieczorek, "Fragmentation -- coagulation models of phytoplankton", to appear in Bull. Pol. Acad. Sci. Math.,

[RW2] R. Rudnicki and R. Wieczorek "Phytoplankton dynamics: from the behaviour of cells to a transport equation", preprint,

[BLPR] A. Bobrowski, T. Lipniacki, K. Pichor, R. Rudnicki, "Asymptotic behavior of distributions of mRNA and protein levels in a model of eukaryotic gene expression" submitted to J. Math. Anal. Appl.

In [RW1] two new models of the dynamics of phytoplankton aggregates are presented. The first one is an individual based model. Passing to infinity with the number of individuals, the authors obtain an Eulerian model. This model describes the evolution of the density of the spatial-mass distribution of aggregates. The authors show the existence and uniqueness of the solutions of the evolution equation.

In [RW2] the authors present models of the dynamics of phytoplankton aggregates. They start with an individual based model in which aggregates can grow, divide, join and move randomly. Passing to infinity with the number of individuals we get a model which describes the space-size distribution of aggregates. The density distribution function satisfies a non-linear transport equation, which contains terms responsible for the growth of phytoplankton aggregates, their fragmentation, coagulation, and diffusion.

[BLPR] is devoted to a stochastic process used in modelling gene expression in eucaryotes. We show that its distributions satisfy a Fokker-Planck-type system of partial differential equations. The authors construct a Markov semigroup corresponding to this system. The main result of the paper is asymptotic stability of the involved semigroup in the set of densities.

2. PDE's

a) T. Reginska in part on her stay at the SPADE2 partner institution CAU Kiel, wrote the paper [RR] T. Reginska and K. Reginski - "A Cauchy problem for the

Helmholtz equation: application to analysis of light propagation in solids" - preprint in *Berichtsreihe des Mathematischen Seminars der Universitat Kiel*. She studied the problem of determination of radiation field inside a solid from experimental data given on a part of surface surrounding this solid is considered. The model problem has been formulated as a Cauchy problem for the Helmholtz equation. For solving it, an approximate method based on regularization in frequency space is formulated and analyzed. Convergence and stability of the method are proved under a suitable choice of regularization parameter and numerical implementation of the method is presented. Possible application of the method to problems of propagation of laser beams in solids is discussed.

b) Several results concern existence of global solutions of Navier-Stokes Equation; obtained in relation to the visit by Konstantin Pileckas. They have been written in:

[P1] K. Pileckas, "Global solvability in $W^{2,1}_2$ -weighted spaces of the two-dimensional Navier-Stokes problem in domains with strip-like outlets to infinity" (submitted to J. of Math. Fluid Mech.).

[PZ1] K. Pileckas, W. M. Zajaczkowski, "Global solvability for a large flux of a three-dimensional time dependent Navier-Stokes problem in a straight cylinder" (submitted to *Studia Mathematica*). In the paper the unique global existence of a solution to nonstationary Navier-Stokes system with prescribed nonzero flux $F(t)$ in an infinite three-dimensional pipe is proved. The obtained

solution remains close to the corresponding nonstationary Poiseuille flow. Moreover, it converges to the Poiseuille flow as $|x_3| \rightarrow \infty$.

[PZ2] K. Pileckas, W. M. Zajączkowski, "Solvability of the Cauchy problem for the Navier-Stokes equations in \mathbb{R}^3 for some class of initial data". In this paper it is proved the existence of regular solutions to the Navier-Stokes equations if the initial data v_0 have some finite weighted norm and $\text{supp } v_0$ belongs to $\mathbb{R}^3 \setminus B_{R_0}$, B_{R_0} is a ball with radius R_0 , where R_0 is sufficiently large. The proof follows from appropriate estimates in weighted Sobolev spaces.

[RZ] In J. Renclawowicz and W. Zajączkowski in "Weak solutions to the Navier-Stokes equations in a Y-shaped domain", *Applicationes Mathematicae*, 33,1 (2006), pp. 111-127, it is proved the existence of weak solutions to the Navier-Stokes equations describing the motion of a fluid in a Y-shaped domain is shown. More precisely, it is the inflow-outflow problem in the reverse Y-shaped domain, with one inflow and two outflows. This can be treated as a simple model of the blood flow in veins or arteries. The motion of the fluid is described by the Navier-Stokes equations with boundary slip conditions. The energy type estimates and the Galekin method were used.

c) In a preprint of Piotr Bogusław Mucha "Stability of 2D incompressible flows in \mathbb{R}^3 ", global stability of solutions of Navier-Stokes equations are studied. A typical example are 2-dimensional flows. The main trouble is an absence of Poincaré's inequality.

d) In [BLP] by G. Łukaszewicz et al. "On pull-back attractors for a class of 2-dimensional turbulent shear flows" the main result concerns the existence proof of the pullback attractor for nonautonomous infinite dimensional dynamical systems lacking compactness property, and the proof of finite dimensionality of the attractor.

Application in studying turbulent flows in unbounded domains (channel like domains) followed.

3. Stochastic Processes, Scaling Limits.

a) M. Zinsmeister prepared notes on SLE, Stochastic (Schramm) Loewner Evolution, available on web page: <http://www.univ-orleans.fr/mapmo/membres/zins/SLE/>. This new theory was created in recent years by Lawler, Schramm and Werner, the latter got in Madrid ICM in August 2006.

b) J. Zabczyk and Z. Priola and also PhD student A. Rusinek prepared papers on stochastic evolution equations.

4. Function Spaces

a) Michał Wojciechowski on his stay in Pisa worked with G. Alberti on several topics connected with the singular measures: Dunford-Pettis property of the Sobolev spaces with L^1 norm, spaces of measures with norm given by symmetric derivatives, Lipschitz projections on the space of smooth functions, approximation property of Sobolev spaces of functions on arbitrary domain. Let us mention his preprint "Bounded approximation property of the spaces of functions of bounded variation".

b) Fruitful was the visit at IMPAN by Alexander Koldobsky. The paper A. Koldobsky, C. Shane "Determination of convex bodies by derivatives of section functions", *Archiv der Math.*, belongs to the area of geometric tomography where one tries to study properties of solids based on the information about their sections and projections. A typical example is x-ray tomography. The classical result of Minkowski shows that a symmetric convex body can be uniquely determined by the areas of central hyperplane sections in all directions. In the non-symmetric case it is not enough to know the areas of sections through one point, but, as shown by Falconer and Gardner, the areas of hyperplane sections through two different interior points determine the body. This result is generalized by proving that if q is not an integer then the derivatives of parallel section functions with respect to two different points determine the body.

Teaching and training activities

The "incoming" researchers gave the following lectures to the young researchers of the IMPAN and students of the Warsaw University.

-Michel Zinsmeister: "Complex analysis, stochastic Loewner evolution and applications" a semester course on PhD students level (24 hours of lectures + exercises)

-P.B. Mucha and W. Zajączkowski: A course "Linear PDEs in L_p -spaces".

-Alexander Koldobsky, some lectures at seminars on his stay in Poland:

Functional Analysis Seminar, IMPAN, "Inequalities of the Kahane-Khinchin type and sections of

L_p -balls" and "Fourier approach to sections and projections of convex bodies".
 Probability Seminar, University of Warsaw, "Embedding of normed spaces in L_p , $p < 0$ ".
 Probability Seminar, Technical University of Wroclaw, "Inequalities of the Kahane-Khinchin type and sections of L_p -balls"
 Graduate Students Seminar, Institute of Mathematics PAN, "Fourier analysis in convex geometry"
 -Konstantin Pileckas gave following lectures:
 A mini-course (4 lectures) at the Faculty of Mathematics of the Warsaw University, "Navier-Stokes equations in pipes"
 SPADE seminaire, "Steady and time dependent Navier-Stokes equations in domains with non-compact boundaries"

Furthermore the SPADE 2 seminar DETERMINISTIC AND STOCHASTIC DYNAMICS, FRACTALS, TURBULENCE was organized and the following workshops and conferences devoted to the subjects close to the field of SPADE 2 were organized:
 Non-Uniform Hyperbolicity in One-Dimensional Dynamics-workshop in Warsaw, April 3-7, 2006 ;
 Fractal Geometry and Dynamics-school in Warsaw, April 10-14, 2006;
 School on Dynamical Systems- school in Warsaw, April 28- May 1, 2006;
 Analysis and Partial Differential Equations - conference in Bedlewo, June 18-23, 2006;
 Parabolic and Navier-Stokes Equations - conference in Bedlewo, September 10-16, 2006;
 Forum of Differential Equations - conference in Bedlewo, May 28- June 02, 2006;
 9th Conference on Probability - conference in Bedlewo, May 22-26, 2006.

Dissemination of results

The following publications were written within the SPADE 2 programme

1. Dynamical Systems

- [R-LV] M. Rams, J. Levy-Vehel, "Results on the Dimension Spectrum for Self-conformal Measures" submitted.
 [RW1] Rudnicki and R. Wieczorek, "Fragmentation -- coagulation models of phytoplankton", Bull. Pol. Acad. Sci. Math. (in press).
 [RW2] R. Rudnicki and R. Wieczorek, "Phytoplankton dynamics: from the behaviour of cells to a transport equation", submitted to: Mathematical Modelling of Natural Phenomena.
 [BLPR] A. Bobrowski, T. Lipniacki, K. Pichor, R. Rudnicki, "Asymptotic behavior of distributions of mRNA and protein levels in a model of eukaryotic gene expression" submitted to: J. Math. Anal. Appl.
 [KS] J.Kotus, G. Swiatek, "Invariant measures for meromorphic Misiurewicz maps".

2. PDE's

- [RR] T. Reginska and K. Reginski - "A Cauchy problem for the Helmholtz equation: application to analysis of light propagation in solids" preprint in *Berichtsreihe des Mathematischen Seminars der Universitat Kiel*.
 [RZ1] J. Renclawowicz, W.M. Zajaczkowski, "Weak solutions to the Navier-Stokes equations in a Y-shaped domain", *Applicationes Mathematicae*, 33,1 (2006), pp. 111-127.
 [BLR] M.Boukrouche, G.Lukaszewicz, J.Real, "On pullback attractors for a class of two-dimensional turbulent shear flows", *International Journal of Engineering Science*, vol.44, pp. 830-844, (2006).
 [M1] P.Mucha, "Stability of 2D incompressible flows in R^3 ", preprint
 [P] K.Pileckas, "Global solvability in $W^{2,1}_2$ -weighted spaces of the two-dimensional Navier-Stokes problem in domains with strip-like outlets to infinity" submitted to: *J. of Math. Fluid Mech.*
 [PZ1] K. Pileckas, W. M. Zajaczkowski, "Global solvability for a large flux of a three-dimensional time dependent Navier-Stokes problem in a straight cylinder" submitted to: *Studia Mathematica*.
 [GPZ] G.P. Galdi, K. Pileckas, A.M. Robertson, "On the unsteady Poiseuille flow in a pipe", submitted to: *Zeitschrift fur Angew. Mathematik und Physik*
 [PZ2] K. Pileckas, W. M. Zajaczkowski, "Solvability of the Cauchy problem for the Navier-Stokes

equations in \mathbb{R}^3 for some class of initial data".

3. Stochastic Processes

- [PZ] E.Priola, J. Zabczyk "On bounded solutions to convolution equations", to appear in PAMS.
[R] A. Rusinek, "Invariant measures for a class of stochastic evolution equations", preprint IMPAN.
[Z1] M. Zinsmeister, D.Grebenkov, K.Kolwankar, P.Levitz, B.Sapoval, "Brownian flights over a fractal nest and first-passage statistics on irregular surfaces", Phys. Rev. Lett. 96 (2006), 180601.
[Z2] M. Zinsmeister, Stochastik Loewner Evolution. Notes to a course in Warsaw, Spring 2006.
<http://www.univ-orleans.fr/mapmo/membres/zins/SLE/>

4. Function Spaces

- [KK] A. Kalamajska i M. Kruzik, Oscillations and concentrations in sequences of gradients, preprint porzyjety w ESAIM: Control, Optimization and Calculus of Variations.
[K1] A. Kalamajska, Lusin type theorem with convex integration and quasicontinuous hulls of sets, preprint.
[K2] A. Kalamajska, Oscillation and concentration effects described by Young measures which control discontinuous functions.
[KP] A. Kalamajska i K. Pietruska-Paluba, Gagliardo-Nirenberg in weighted Orlicz spaces equipped with a not necessarily doubling measure.
[Ko1] A.Koldobsky, C.Shane "Determination of convex bodies by derivatives of section functions", Archiv der Math., to appear
[Ko2] A.Koldobsky, A.Pajor, V.Yaskin "Khinchin type inequalities and sections of L_p -balls", preprint
[W] M. Wojciechowski, "Bounded approximation property of the spaces of functions of bounded variation".

In Annex I we list attendance of conferences and invited university lectures by participants of the programme, where some results related to the programme were presented.

ADDITIONAL INFORMATION

Please indicate any additional information, which may be considered useful to assess the work done during the reporting period. The socio-economic aspect of the project may be addressed in this section.

The Polish Ministry of Science and Higher education awarded the project a matching grant of about 450 000 euros. This grant has been used to support short visits at IMPAN of foreign and Polish (from other cities) mathematicians and also short outgoing visits, substantial for the success of the program, supports schools and workshops, enables us also to buy some necessary computers, programs etc.

Attachments	Annex I.doc
Name	
Date	
Signature	