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On the class of Brill-Noether loci for Prym varieties

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Summary. In this note we give a formula for the cohomology class of the Brill Noether loci V^r introduced by Welters for Prym varieties in [9]. The strategy of our computation is similar to the one used by Kempf and Kleiman-Laksov (see [1, Chap.VII]) to compute the cohomology class of classical Brill-Noether loci in Jacobians. The main difference is that using a result of Mumford [6] our computation can be reduced to the computation of the class of a suitable degeneracy locus associated with a pair of maximal isotropic subbundles in a vector bundle endowed with a non degenerate quadratic form. For this we can use formulas from [7] and [8]. In particular our method implies the non-emptiness of the V^r, in the range $g \geq \binom{r+1}{2} + 1$, thus giving another proof of the existence theorem for V^r conjectured by Welters in [9] and first proved by Bertram in [2] using different methods.

Let k be an algebraically closed field of characteristic different from 2. Let $\pi : \tilde{C} \to C$ be an étale double cover of a smooth algebraic curve C over k of genus $g = g(C)$. Then $g(\tilde{C}) = 2g - 1$ and we have a norm map $Nm : Pic^{2g-2}(\tilde{C}) \to Pic^{2g-2}(C)$. We consider the scheme $Nm^{-1}(\omega_C)$, ω_C being the canonical class. This scheme has two connected components P^+ and P^- depending on the parity of $h^0(-)$ [9]. We recall the definition of the loci $V^r = V^r(C, \pi)$ from [9, Def.(1.2)]. For every integer $r \geq -1$, we set

$$V^r = \{L \in P^{\pm} | h^0(L) \geq r+1 \text{ and } h^0(L) \equiv r+1 (mod 2)\}.$$

Of course $V^r \subset P^+$ (resp. $V^r \subset P^-$) iff r is odd (resp. even).

Let $\mathcal{L}$ be a Poincaré line bundle on $Pic^{2g-2}(\tilde{C}) \times \tilde{C}$. Consider the double cover $1 \times \pi : Pic^{2g-2}(\tilde{C}) \times \tilde{C} \to Pic^{2g-2}(\tilde{C}) \times C$. Let $\mathcal{E} = (1 \times \pi)_*(\mathcal{L})$. Then by the construction in [6, p.185], if we restrict the rank 2 vector bundle $\mathcal{E}$ to $P^{\pm} \times C$, we get a rank 2 vector bundle which is endowed with a non degenerate quadratic form (corresponding to the form "Q" in the notation of loc.cit.) with values in the restriction to $P^{\pm} \times C$ of the invertible sheaf $q^*(\omega_C)$, $q : Pic^{2g-2}(\tilde{C}) \times C \to C$ being the projection on the second factor.

Let D be an effective divisor of sufficiently positive degree on C. Consider the projection $p : Pic^{2g-2}(\tilde{C}) \times C \to Pic^{2g-2}(\tilde{C})$ and let $V = p_*(\mathcal{E}(D)/\mathcal{E}(-D))$, $W = p_*(\mathcal{E}(D))$ and $U = p_*(\mathcal{E}/\mathcal{E}(-D))$. Due to the very positive degree of D we have that V , W and U are locally free sheaves and we shall denote by the same letters the corresponding vector bundles. Furthermore (see [6, p.183] both W and U are in a natural way subbundles of V. Take now $V_{P^{\pm}}$ (resp.$W_{P^{\pm}}$, $U_{P^{\pm}}$) to be the restriction to $P^{\pm}$ of V (resp. W, U). Then, by [6, p.184], $V_{P^{\pm}}$ is endowed with a non degenerate quadratic form (corresponding to the

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form "q" in the notation of loc.cit.) with values in $\mathcal{O}_{P^\pm}$. The bundles $W_{P^\pm}$ and $U_{P^\pm}$ are maximal isotropic subbundles of $V_{P^\pm}$ with respect to this form.

If we fix now a line bundle L of degree $2g-2$ on $\tilde{C}$ and let $E = \pi_*(L)$ we get [6, p.183] that $\Gamma(\tilde{C}, L) = \Gamma(C, E) = \Gamma(C, E(D)) \cap \Gamma(C, E/E(-D))$ as subspaces in $\Gamma(C, E(D)/E(-D))$. This clearly defines our locus V^r as the locus of points in $P^\pm$ where the two subbundles $W_{P^\pm}$ and $U_{P^\pm}$ intersect fiberwise in dimension at least $r+1$. In particular this endows the loci V^r with a canonical structure of a subscheme of $P^\pm$ (induced by the subscheme structure of the appropriate Schubert subschemes in the orthogonal Grassmannian).

To give a general formula for the class of V^r in terms of Chern classes of the various bundles involved we shall first put ourselves in a general situation.

Let X be a smooth variety and let E be a vector bundle over X. Let c_iE be its i-th Chern class. We set $P_iE = \frac{c_iE}{2}$ and for $i > j$, $P_{i,j}E = P_iE \cdot P_jE + 2\sum_{p=1}^{j-1}(-1)^p P_{i+p}E \cdot P_{j-p}E + (-1)^j P_{i+j}E$. In general, for a strict partition $I = (i_1 > \ldots > i_l)$, l even (by putting $i_l = 0$ if necessary), we set P_IE equal to the Pfaffian of the antisymmetric matrix with $P_{i_p,i_q}E$ as (p,q)-entry for $p < q$.

We now record the following result (extracted from [7] and [8]) which is accompanied by a proof for the reader's convenience.

1. Proposition. *Let $\mathcal{V} \to X$ be a rank $2n$ vector bundle over X, endowed with a non degenerate quadratic form. Let $\mathcal{W}$ and $\mathcal{U}$ be two rank n isotropic subbundles of $\mathcal{V}$. There exists a polynomial $\mathcal{P}(c_1, ..., c_n, c_1', ..., c_n')$ with the following properties:*

1. If the locus

$$\{x \in X | dim(\mathcal{W}_x \cap \mathcal{U}_x) \geq r+1 \text{ and } dim(\mathcal{W}_x \cap \mathcal{U}_x) \equiv r+1 (mod 2)\}$$

is either empty or of codimension $\binom{r+1}{2}$, then $\mathcal{P}(c.\mathcal{W}^*, c.\mathcal{U}^*)$ evaluates its fundamental class in the Chow ring.

2. Under the substitution $c_1' = \ldots = c_n' = 0$ and $c_i := c_i\mathcal{W}^*$, $1 \leq i \leq n$; the value of this polynomial becomes $P_{\rho_r}\mathcal{W}^*$ where ρ_r is the partition $(r, r-1, \ldots, 1)$.

Proof. We prove first the assertion 1 for the locus D defined as follows. For $\mathcal{V} \to X$ as above let $\mathcal{U}$ be an isotropic rank n subbundle of $\mathcal{V}$, $\mathcal{W}$ be the tautological subbundle on the Grassmannian G of such subbundles where $rank(\mathcal{W} \cap \mathcal{U}) \equiv r+1 (mod 2)$ and D consists of those points in G where $rank(\mathcal{W} \cap \mathcal{U}) \geq r+1$.

Let $\nu : F \to X$ be the flag scheme of pairs of subbundles $\mathcal{A} \subset \mathcal{B}$ of ranks $r+1, n$ in $\mathcal{V}$ such that $\mathcal{A} \subset \mathcal{U}$, $\mathcal{B}$ is isotropic and $rank(\mathcal{U} \cap \mathcal{B}) \equiv r+1 (mod 2)$. Let $\mathcal{D}$ be the rank n tautological bundle on F. The map $\alpha : F \to G$ defined by $\alpha(\mathcal{A}, \mathcal{B}) = \mathcal{B}$, i.e. such that $\alpha^*\mathcal{W} = \mathcal{D}$, induces a section $\sigma : F \to GF := G \times_X F$ of the projection on F. Then $Z := \sigma(F)$ is a desingularization of D. The class of Z is the image via $(1 \times \alpha)^*$ in the Chow ring $A(GF)$ of the class of the diagonal of $G \times_X G$.

It follows from [7, Thm 6.17'] that $A(G)$ is a free $A(X)$-module with a basis given by $P_I\mathcal{W}^*$ (for all strict partitions $I \subseteq \rho_{n-1}$) that correspond to Schubert cycles. Given a strict partition $I \subset \rho_k$, let $\rho_k \setminus I$ denote the strict partition whose parts complement the parts of I in $\{k, k-1, \ldots, 1\}$. Since (over a point) the Poincaré dual to $P_I\mathcal{W}^*$ is $P_{\rho_{n-1}\setminus I}\mathcal{W}^*$ (see loc. cit.), one gets that the class of the above diagonal in $A(GF)$ is $\sum P_I\mathcal{W}^*_{GF}\cdot P_{\rho_{n-1}\setminus I}\mathcal{D}^*_{GF}$ (the sum over all strict $I\subset\rho_{n-1}$). Denoting by $\eta : G\times_X F\to G$ the projection we have

$$[D]=\eta_*[Z]=\sum P_I\mathcal{W}^*\cdot\eta_*(P_{\rho_{n-1}\setminus I}\mathcal{D}^*_{GF}).$$

To compute $\eta_*P_J\mathcal{D}^*_{GF}$ it suffices to calculate $\nu_*P_J\mathcal{D}^*$ and use a base change. Recall that $\nu : F\to X$ is a composition of a Grassmannian bundle $G'=G_{r+1}(\mathcal{V})$ and the Grassmannian of rank $(n-r-1)$-isotropic subbundles in $\mathcal{V}_{G'}/(\mathcal{C}\oplus\mathcal{C}^*)$ where $\mathcal{C}$ is the tautological bundle on G' and $\mathcal{C}^*$ is its dual "materialized" in $\mathcal{V}_{G'}$ with the help of the quadratic form (which then induces a form on $\mathcal{V}_{G'}/(\mathcal{C}\oplus\mathcal{C}^*)$). For such Grassmannian bundles one has formulas asserting that the associated Gysin push forward of a polynomial in the Chern classes of the tautological bundle is a polynomial in the Chern classes of the initial bundle (for the usual Grassmannian such a formula is e.g. recalled in [7, Sect.3], for the orthogonal case such a formula is given in [8]). By (the existence of) those formulas we get that $\nu_*P_J\mathcal{D}^*$ is a polynomial in the Chern classes of $\mathcal{U}^*$ and $\mathcal{V}^*(\simeq\mathcal{U}^*\oplus\mathcal{U})$, which implies the existence of polynomial $\mathcal{P}$ such that $[D]=\mathcal{P}(c.\mathcal{W}^*,c.\mathcal{U}^*)$. (In general, by the assumptions of the assertion 1 and standard arguments, there exists a morphism $s : X\to G$ such that the class of the locus in question is $s^*[D]$ i.e. it is given by $\mathcal{P}$.) This proves the assertion 1.

Specializing X to a point we get by [7, Thm 6.17'] applied to G that $[D]=P_{\rho_r}\mathcal{W}^*$; this implies the assertion 2. $\square$

2. Remark. The above result and the method of its proof stems from [8] where we refer for more details and for the following precise formula. If the locus considered in the proposition is either empty or of codimension $\binom{r+1}{2}$, then its class in the Chow ring equals $\sum P_I\mathcal{W}^*\cdot P_{\rho_r\setminus I}\mathcal{U}^*$, the sum over all strict $I\subset\rho_r$.

We want to apply the proposition to the above situation so, from now on, the role of the bundles $\mathcal{V}$, $\mathcal{W}$ and $\mathcal{U}$ will be played by $V_{P\pm}$, $W_{P\pm}$ and $U_{P\pm}$

Before stating the next lemma we need two comments. Since the proof of the lemma will use the Poincaré formula (see [1, p.25], for instance), we must specify the cohomology theory where this formula holds. These are : the singular cohomology with coefficients in $\mathbb{C}$ for the ground field $k=\mathbb{C}$ (see loc.cit.), and the numerical equivalence ring for an arbitrary ground field k (see [5]). In those cohomology rings will be located the Chern classes in the next lemma. Note that, in general, one can replace here the numerical equivalence neither by the rational equivalence nor by the algebraic one. This is because, by [3, Cor.(3.13)], the Poincaré formula is not valid in the algebraic equivalence ring of a Jacobian of a generic curve. We are indebted to A.Collino for this reference.

Moreover, assuming the definition (see e.g. [1, Chap.I.3]) of the theta divisor on the Jacobian of a curve C (which is canonically isomorphic to $Pic^0(C)$), we will understand by the theta divisor on $Pic^n(C)$ the image of the theta divisor on $Pic^0(C)$ under the translation $a : Pic^0(C)\to Pic^n(C)$ defined by $a(L)=L\otimes\mathcal{O}(D)$ where D is a divisor of degree n. Obviously the cohomology class of this theta divisor on $Pic^n(C)$ does not depend on the choise of D. Note that this convention was assumed in the calculations in [1, pp.318-319] which we will use in the following lemma.

3. Lemma. *For sufficiently positive D one has*

1) $c_i(U_{P^\pm}) = 0$ *for* $i > 0$,

2) $c_i(W^*_{P^\pm}) = \frac{\Theta'^i}{i!}$ *where* Θ' *is the restriction to* $P^\pm$ *of the class* Θ *of the theta divisor on* $Pic^{2g-2}(\tilde{C})$.

Proof. 1) In [1, p.309] it is shown that if $\tilde{D}$ is an effective divisor on $\tilde{C}$ of sufficiently positive degree and $\tilde{p} : Pic^{2g-2}(\tilde{C}) \times \tilde{C} \to Pic^{2g-2}(\tilde{C})$ is the projection on the first factor, then $\tilde{p}_*(\mathcal{L}/\mathcal{L}(-\tilde{D}))$ has vanishing Chern classes. But $U = p_*(\mathcal{E}/\mathcal{E}(-D)) = \tilde{p}_*(\mathcal{L}/\mathcal{L}(-\pi^*D))$. It follows that U has vanishing Chern classes, hence also $U_{P^\pm}$ has, and the assertion 1) follows.

2) We have $p_*(\mathcal{E}(D)) = \tilde{p}_*(\mathcal{L}(\pi^*D))$. Let $a : Pic^{2g-2}(\tilde{C}) \to Pic^{2g-2+d}(\tilde{C})$ be the translation by the divisor class of π^*D where $d = degD$. Then $\mathcal{L}(\pi^*D) = (a \times 1)^*(\mathcal{L}_1)$ where $\mathcal{L}_1$ is a Poincaré line bundle on $Pic^{2g-2+d}(\tilde{C}) \times \tilde{C}$. By [1, pp.318-319] applied to the projection $r : Pic^{2g-2+d}(\tilde{C}) \times \tilde{C} \to Pic^{2g-2+d}(\tilde{C})$ and the Poincaré line bundle $\mathcal{L}_1$, we get that $c(r_*(\mathcal{L}_1)) = e^{-\Theta_1}$ where Θ_1 is the class of the theta divisor on $Pic^{2g-2+d}(\tilde{C})$. Since at this point we use the Poincaré formula, for the numerical equivalence ring we must invoke [5, Formula (4) p.84]. Since $a\tilde{p} = r(a \times 1)$, we infer that $\tilde{p}_*(a \times 1)^*(\mathcal{L}_1) = a^*r_*(\mathcal{L}_1)$. Consequently we have

$$c(W) = c(p_*(\mathcal{E}(D))) = c(\tilde{p}_*(\mathcal{L}(\pi^*D))) = c(\tilde{p}_*(a \times 1)^*(\mathcal{L}_1)) =$$

$$c(a^*r_*(\mathcal{L}_1)) = a^*(c(r_*(\mathcal{L}_1))) = a^*(e^{-\Theta_1}) = e^{-\Theta}.$$

Restricting to $P^\pm$, the assertion 2) follows. □

By this Lemma and Proposition 1 we then immediately get that the class of V^r is $P_{\rho_r}(W^*_{P^\pm})$. Then by the given formula for $c(W^*_{P^\pm})$ we obtain that $P_{\rho_r}(W^*_{P^\pm}) = \Theta'^{\binom{r+1}{2}}\alpha$, where α is a rational number. In order to compute α more explicitly (in particular to show that $\alpha \neq 0$) we need some preliminary considerations.

Let Q_i $i = 1, 2, \ldots$ be a sequence of variables. We define $Q_{i,j}$, for $i > j$, by

$$Q_{i,j} = Q_iQ_j + 2\sum_{p=1}^{j}(-1)^p Q_{i+p}Q_{j-p}.$$

More generally, for a strict partition $I = (i_1 > \ldots > i_l)$, l even (by putting $i_l = 0$ if necessary), we set Q_I equal to the Pfaffian of the antisymmetric matrix with Q_{i_p,i_q} as (p,q)-entry if $p < q$. We then state:

4. Proposition. *Under the specialization* $Q_i := \frac{1}{i!}$, *one has for a strict partition* $I = (i_1, \ldots, i_l)$,

$$Q_I = \frac{1}{i_1!i_2!\cdots i_l!}\prod_{p<q}\frac{i_p - i_q}{i_p + i_q}.$$

Proof. For Q_i the formula is obvious. We prove it for $Q_{i,j}$ by induction on j. Since, by the definition of $Q_{i,j}$, we have

$$Q_{i,j} = Q_i Q_j - Q_{i+1} Q_{j-1} - Q_{i+1,j-1},$$

the assertion follows from the identity

$$\frac{1}{i!j!}\frac{i-j}{i+j} = \frac{1}{i!}\frac{1}{j!} - \frac{1}{(i+1)!}\frac{1}{(j-1)!} - \frac{1}{(i+1)!(j-1)!}\frac{i-j+2}{i+j},$$

a verification of which is straightforward.

In the general case we can assume l even (by putting $i_l = 0$ if necessary). After our substitution the antisymmetric matrix used in the definition of Q_I is the product of matrices ABA where A is the diagonal matrix having $\frac{1}{i_p!}$ as its (p,p)-entry while B is the antisymmetric matrix whose (p,q)-entry equals $\frac{i_p - i_q}{i_p + i_q}$. It is well known that the Pfaffian of B equals

$$\prod_{p<q} \frac{i_p - i_q}{i_p + i_q},$$

which, together with $Q_I = Det(A) \cdot Pf(B)$, implies the claimed formula. □

5. Remark. The above given formula for Q_I can be expressed combinatorially as the inverse of the product of all hook lengths of boxes in the shifted diagram associated with I embedded in the shift symmetric diagram of I (i.e. the diagram of the partition $(i_1, \ldots, i_l; i_1 - 1, \ldots, i_l - 1)$ in Frobenius' notation).

We are now ready to compute α.

6. Lemma. *The rational number α defined above equals*

$$2^{-r} \prod_{i=1}^{r} \frac{(i-1)!}{(2i-1)!}.$$

Proof. For a vector bundle E it is immediate from the definitions that if we specialize $Q_i := c_i E$, we obtain that $P_I E = 2^{-\ell(I)} Q_I$, where $\ell(I)$ is the number of non zero parts in I. Applying this identity to the bundles $W^*_{P\pm}$ and using the expression:

$$Q_{\rho_r} = \prod_{i=1}^{r} \frac{1}{i!} \prod_{1 \le j < i \le r} \frac{i-j}{i+j},$$

given by Proposition 4, we get the asserted formula by an easy induction on r. Observe that the Lemma follows also from the hook length interpretation of Proposition 4 mentioned in Remark 5. Note that this remark gives an alternative expression for α in the form $2^{-r} \prod_{i=0}^{r-1} \frac{(2i)!}{(r+i)!}$. □

We can summarize the above considerations in the following theorem.

7. Theorem. *Assume that either V^r is empty or has pure codimension $\binom{r+1}{2}$ in $P^{\pm}$. Then its fundamental class in the numerical equivalence ring of $P^{\pm}$ (for an arbitrary ground field k with $char(k) \neq 2$), or the cohomology dual to its fundamental class in $H_*(P^{\pm}, \mathbb{Z})$ (for $k = \mathbb{C}$), is equal to*

$$2^{-r} \prod_{i=1}^{r} \frac{(i-1)!}{(2i-1)!} \Theta'^{\binom{r+1}{2}},$$

where Θ' is the restriction to $P^{\pm}$ of the class of the theta divisor on $Pic^{2g-2}(\tilde{C})$.

In particular, if $g \geq \binom{r+1}{2} + 1$, then V^r is not empty of dimension at least $g - 1 - \binom{r+1}{2}$.

Proof. Everything follows from our previous computations once we recall that the theta divisor on $Pic^{2g-2}(\tilde{C})$ (and thus also its restriction to $P^{\pm}$) is ample. □

8. Remark. 1) The last assertion of the theorem i.e. the existence theorem for V^r was proved originally by Bertram in [2], as C.Keem has pointed out to us. Our approach offers a new proof of this result.

2) In the case of the Prym variety P^+ we know that $\Theta' = 2\Xi$, Ξ being the class of the theta divisor on P^+. Thus, if the reader wishes he can express our formula in terms of Ξ. After the first version of this note was written, we have learned from Welters that a particular case of our formula for P^+, namely the case r=3, was a subject of [4, Théorème 1.1(i)] whose derivation used methods quite different from ours.

3) In [9] Welters showed that if C is a general curve and if $\pi : \tilde{C} \to C$ is any irreducible étale double cover, then V^r, if non-empty, has indeed pure codimension $\binom{r+1}{2}$.

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