Friedrich Hirzebruch – a handful of reminiscences*

Piotr Pragacz**

The idea of dedicating an IMPANGA volume to Friedrich Hirzebruch arose a few years ago. A lot has changed since then. First of all, Friedrich Hirzebruch passed away on May 27, 2012. Following his death many conferences, lectures and articles [2, 4, 3, 22] were dedicated to him. The articles [1, 5, 7, 9] and the video-interview [13] appeared while he was still alive. A book by Winfried Scharlau is in preparation. These publications accurately describe the life and work of Professor Friedrich Hirzebruch from the point of view of his close colleagues and coworkers. Therefore, though initially I intended to write an article about him similar to *Notes on the life and work of Alexander Grothendieck* [16] (1st volume of IMPANGA Lecture Notes) or the one on Józef Maria Hoene-Wroński [17] (2nd volume of IMPANGA Lecture Notes), I decided to change my plans, so this essay will be of a different nature. I would like to share a collection of reminiscences about Professor Friedrich Hirzebruch from the vantage of a person, for whom he was a mentor in the years 1993–2006. In addition, I would like to highlight his relations with IMPANGA.

F. Hirzebruch during the Leonhard Euler Congress in Saint Petersburg in 2007

I met Friedrich Hirzebruch for the first time in 1988 during the algebraic semester at the Banach Center, at the old residence on Mokotowska street. He came for the conference in algebraic geometry organized as a part of that semester. The

*This article was translated by Masha Vlasenko with the editorial assistance of the author. He also made a couple of additions. The article originally appeared as [18] in the Polish journal *Wiadomości Matematyczne*, and we thank the Editors of this journal for permission to publish this English translation.

**Supported by National Science Center (NCN) grant no. 2014/13/B/ST1/00133

main organizer was Wolfgang Vogel. I was helping with organization. Professor Hirzebruch's lecture was a highly emotional experience for us, the younger participants. He was speaking about links between algebraic geometry and physics. It was a lecture of a Master. I also gave a talk at that conference. I spoke on polynomials in Chern classes supported on determinantal varieties. Professor Hirzebruch was present at my talk and asked a couple of interesting questions. I had an impression that he liked what I was doing.

In 1992 Adam Parusiński encouraged me to apply, following himself, for a scholarship from the Humboldt Foundation. For that reason I had to visit the Max Planck Institute in Bonn for two weeks. The main purpose of the visit was my talk at the Oberseminar, where I presented a development of my theory of polynomials supported on degeneracy loci. I recall that besides Professor Hirzebruch in the audience there were Robert MacPherson, Christian Okonek, Don Zagier and others. I had an impression that my talk was met with a positive reception. Shortly after I received the Humboldt Scholarship under which I spent years 1993–1995 at the Max Planck Institute in Bonn, where my host professor was Professor Friedrich Hirzebruch. During that visit Professor Hirzebruch became my mentor, and remained so in the following years. To be honest, it was already after my habilitation, but it was Professor Hirzebruch who helped me to find my place in mathematics.

Let us be more systematic. We will start with a short biography.

Friedrich Hirzebruch was born on October 17, 1927 in Hamm in North Rhine-Westphalia. His father was a teacher of mathematics and director of a gymnasium. He was also a teacher of the famous mathematician Karl Stein (the terms *Stein space* and *Stein factorization* are named after him).

In the years 1945–48 Friedrich Hirzebruch studied mathematics at the University of Münster, which was greatly affected by the war. There he mostly studied complex analysis under the guidance of Heinrich Behnke. Hans Grauert and Reinhold Remmert were also alumni of the Münster School. The school had connections with Henri Cartan in Paris.

In the years 1949–50 Friedrich Hirzebruch studied topology under Heinz Hopf in Zurich. There he prepared a doctoral thesis on the resolution of singularities of 2-dimensional complex spaces. Working with Hopf and Beno Eckmann in Zurich, he encountered for the first time Stiefel–Whitney and Chern classes in topology. Chern classes accompanied him for his entire life, see [12]. Studying in Zurich, Hirzebruch realized that results of algebraic topology and algebraic geometry could be applied in order to progress in complex analysis. The stay in Zurich was also fruitful in establishing his first international contacts.

Friedrich Hirzebruch defended his doctoral thesis (advisers: Behnke and Hopf) in Münster in 1950.

He spent the years 1950–1952 at the University of Erlangen.

In the years 1952–1954 he visited the Institute of Advanced Study (IAS) in Princeton. The visit greatly affected his research and mathematical mastery. Shortly upon arrival, Hirzebruch established a collaboration with Kunihiko Kodaira and Donald C. Spencer, and also with Armand Borel. He was corresponding with Jean-Pierre Serre and René Thom. He studied algebraic geometry: sheaves, bundles, sheaf cohomology, Chern classes, characteristic classes, and also cobordism theory. In Princeton Hirzebruch achieved a turning point in his work on the signature theorem and on the Riemann–Roch theorem for projective manifolds of arbitrary dimensions (in December 1953).

The IAS in Princeton enchanted him. Hirzebruch started to cherish the idea of creating something similar in his homeland. He succeeded in 1980, when the Max Planck Institute for Mathematics (MPIM) was established in Bonn. Naturally, Hirzebruch became the first director of that institute.

On returning to Münster in 1954 he wrote the book *Neue topologische Methoden in der algebraischen Geometrie.* The book contained a complete proof of the Riemann–Roch theorem and constituted his habilitation thesis (see [10]).

Hirzebruch visited Princeton once again in 1955/56, this time hosted by the University. During that visit he met Atiyah, Borel, Bott, Chern, Lang, Milnor, Serre, Singer and many others. He also lectured on his habilitation thesis.

In 1956 he was made a professor at the university of the beautiful town of Bonn.

Bonn seen from Venusberg; picture received as a gift from the *Bouvier* bookshop in Bonn

Hirzebruch worked at the University of Bonn until 1993. Though he traveled a lot around the world, there was only one period of long absence: the sabbatical (research leave) at Princeton in 1959/60.

Hirzebruch's favorite mathematical objects were: manifolds, surfaces, singularities (his "first love") and characteristic classes.

Manifolds: topological, algebraic, complex (though his last lecture was about real curves). Many results were obtained in cooperation with Michael Atiyah who was a close friend of Hirzebruch (cf. [22, Sect. 9]).

Surfaces: doctoral thesis devoted to the resolution of singularities of 2-dimensional complex spaces with the help of Hopf's σ-processes; Hilbert modular surfaces were studied jointly with Don Zagier, Antonius Van de Ven and Gerard van der Geer.

Singularities: exotic structures on manifolds arising from singularities studied jointly with Egbert Brieskorn (see [7]).

Characteristic classes: Stiefel–Whitney classes, Chern classes (Shiing-Shen Chern was a close friend of Hirzebruch, see [12]), Pontryagin classes (important in the signature theorem) and the Todd genus (crucial for the Riemann-Roch theorem). Hirzebruch wrote three papers jointly with Borel on characteristic classes of homogeneous spaces. Divisibility of Chern classes and polynomials of Chern classes were among the topics which fascinated Hirzebruch. At the beginning of his study of Chern classes, Hirzebruch was intrigued by the fact that for a smooth algebraic surface $c_1^2 + c_2$ was always divisible by 12. Similarly: $c_n(E)$, where E is a bundle over the $2n$-dimensional sphere, is divisible by $(n-1)!$. At the Arbeitstagung in 1958 he called his talk

"c_n divisible by $(n-1)!$ over the $2n$-sphere"

(cf. [5]).

In 1953 in Princeton Hirzebruch discovered and proved two fundamental theorems: on the signature and Riemann–Roch formula in arbitrary dimension. Let E be a vector bundle of rank r over a (smooth) manifold X. Let $x_1, \ldots, x_r$ be the Chern roots of E. The series

$$\prod_{i=1}^{r} \frac{\sqrt{x_i}}{\tanh \sqrt{x_i}}$$

is symmetric in $x_1, \ldots, x_r$, and therefore it is a series L in the Chern classes $c_1(E), \ldots, c_r(E)$. This series $L(E) = L(c_1(E), \ldots, c_r(E))$ is called the Hirzebruch L-class of E. When E is a real vector bundle over a differentiable manifold X, then *Pontryagin classes* are defined as

$$p_i(E) = (-1)^i c_{2i}(E \otimes \mathbb{C}) \in H^{4i}(X, \mathbb{Z}) \quad \text{for } i = 1, 2, \ldots .$$

If X is a closed oriented manifold of even dimension n, then the intersection form on $H^{n/2}(X)$ is nondegenerate due to Poincaré duality and it is symmetric when n is divisible by 4. Then we define $\sigma(X)$ (the *signature* of X) as the signature of this form, that is the number $p - q$ if the form can be written as

$$z_1^2 + \cdots + z_p^2 - z_{p+1}^2 - \cdots - z_{p+q}^2$$

for a basis $z_1, \ldots, z_{p+q}$ in $H^{n/2}(X, \mathbb{R})$. We refer the reader to [8] for an excellent account to signatures in algebra and topology.

Hirzebruch's signature theorem tells us that for a compact oriented (smooth) manifold X of dimension $4n$ one has

$$\sigma(X) = \int_X L(p_1(X), \ldots, p_n(X)),$$

where $p_i(X) = p_i(TX)$ are called the *Pontryagin classes of* X and TX is the tangent bundle of X.

For example,

$$\begin{aligned}
\sigma(X) &= \frac{1}{3}\int_X p_1(X)\,, \text{ when } \dim X = 4\,,\\
\sigma(X) &= \frac{1}{45}\int_X p_2(X) - p_1(X)^2\,, \text{ when } \dim X = 8\,,\\
\sigma(X) &= \frac{1}{945}\int_X 62p_3(X) - 13p_2(X)p_1(X) + 2p_1(X)^3\,, \text{ when } \dim X = 12\,.
\end{aligned}$$

An important role in the proof of this theorem is played by Thom's results on cobordism. The expository article [11] tells about this and also about the history of the discovery of the Riemann–Roch theorem in higher dimensions.

Now let X be a (smooth) projective complex algebraic manifold of dimension n and $E \to X$ be a vector bundle of rank r. In a letter to Kodaira and Spencer, Serre conjectured that the *Euler number*

$$\chi(X, E) = \sum_i (-1)^i \dim H^i(X, E)$$

can be expressed through the Chern classes of X and E. This was known for curves. Namely, André Weil showed that for a smooth curve X of genus g

$$\chi(X, E) = \int_X c_1(E) + r(1 - g)\,.$$

More explicitly, when $E = \mathcal{O}(D)$ is the line bundle attached to a divisor D, then

$$\chi(X, \mathcal{O}(D)) = \deg D + 1 - g\,,$$

which is the classical Riemann–Roch formula for linear systems on curves.

Many mathematicians tried to generalize these formulas to varieties of higher dimensions. Max Noether, Guido Castelnuovo, Francesco Severi and Oscar Zariski obtained some results for surfaces. Hieronymus G. Zeuthen and Corrado Segre generalized the notion of genus.

A crucial step was made by Hirzebruch. He had at his disposal ingenious computations of John A. Todd, an outstanding British algebraic geometer (he was the advisor of Michael Atiyah). In the 1930s, Todd determined polynomials in the Chern classes of manifolds which give the arithmetic genus for the cases of dimension $d \leq 6$. The arguments consisted essentially of inverting an $N \times N$ matrix, when N is the number of partitions of d. Using these partial results, Todd also verified for $d \leq 6$ the multiplicative property of the arithmetic genus, and conjectured that it should hold more generally. Hirzebruch understood how to generalize the computations of Todd, and defined the *Todd genus*:

$$\prod_{i=1}^{r} \frac{x_i}{1 - e^{-x_i}} = \mathrm{td}(c_1(E), \ldots, c_r(E)) = \mathrm{td}(E)\,.$$

And this Todd genus appeared to be the key to the proof of the Serre conjecture. (Hirzebruch reports in [10, p. 16] that the Todd polynomials, as formal algebraic pieces, appeared already in the work of Nörlund (1924), and were called *Bernoulli polynomials of higher order.*) In [10, 11] Hirzebruch notices that the only power series $f(x)$ with the constant term 1 that satisfies the condition

for every m, *the coefficient of* x^m *in* $f(x)^{m+1}$ *is equal to* 1

is the series

$$f(x) = \frac{x}{1-e^{-x}} = 1 + \frac{x}{2} + \sum_{k=1}^{\infty} B_{2k} \frac{x^{2k}}{(2k)!}\,,$$

where B_{2k} are the *Bernoulli numbers*:

$$B_2 = \frac{1}{6}\,, \quad B_4 = -\frac{1}{30}\,, \quad B_6 = \frac{1}{42}\,, \ldots\,.$$

(In [10, 11] a different notation for the Bernoulli numbers was used: $B_1 = \frac{1}{6}, B_2 = \frac{1}{30}, B_3 = \frac{1}{42}, \ldots$). This characterization was helpful to define the Todd genus. With $c_i = c_i(E)$ we have

$$\mathrm{td}(E) = 1 + \frac{1}{2}\,c_1 + \frac{1}{12}(c_1^2 + c_2) + \frac{1}{24}\,c_1c_2 + \cdots\,.$$

The Hirzebruch–Riemann–Roch theorem (HRR) tells us that

$$\chi(X, E) = \int_X \mathrm{ch}(E)\mathrm{td}(X)\,, \tag{1}$$

where $\mathrm{ch}(E) = \sum_{i=1}^{r} e^{x_i}$ is the *Chern character*, and $\mathrm{td}(X) = \mathrm{td}(TX)$ means the Todd genus. We then have

$$\mathrm{ch}(E) = r + c_1 + \frac{1}{2}(c_1^2 - 2c_2) + \frac{1}{6}(c_1^3 - 3c_1c_2 + 3c_3) + \cdots\,.$$

Write $t_i = c_i(TX)$. If $\dim(X) = 2$, then (1) reads

$$\chi(X, E) = \frac{1}{2}\int_X c_1^2 - 2c_2 + t_1c_1 + \frac{r}{6}(t_1^2 + t_2)\,.$$

For the line bundle $E = \mathcal{O}(D)$ attached to a divisor D, we get

$$\chi(X, \mathcal{O}(D)) = \frac{1}{2}\int_X [D]^2 + t_1[D] + \frac{1}{6}(t_1^2 + t_2)\,.$$

If $\dim(X) = 3$, then (1) reads

$$\chi(X, E) = \int_X \frac{1}{6}(c_1^3 - 3c_1c_2 + 3c_3) + \frac{1}{4}t_1(c_1^2 - 2c_2) + \frac{1}{12}(t_1^2 + t_2)c_1 + \frac{r}{24}t_1t_2\,.$$

The nature of the left-hand side in (1) is holomorphic, while the right-hand side is of a topological nature. We give a sketch of proof from Hirzebruch's book [10]. Let us introduce the notation

$$\mathsf{T}(X,E) = \int_X \mathrm{ch}(E)\mathrm{td}(X)\,.$$

We denote by $\pi : F(E) \to X$ the bundle of flags of subspaces of dimensions $1,\dots,r-1$ in the fibers of E. The bundle $F(E)$ is often called the *bundle of complete flags associated to* E. If X is a point, and so $E = \mathbb{C}^r$, then we get a nonsingular projective variety $F(r)$ of complete flags in $\mathbb{C}^r$. The study of properties of the flag bundle $F(E)$ plays a key role in Hirzebruch's proof. One has ([10, Theorem 14.3.1])

$$T(X,E) = T(F(E),\pi^*E)\,.$$

Let $\chi(X) = \chi(X,\mathcal{O}_X)$ be the *arithmetic genus* (cf. [10, p. 123 and pp. 1–2]). It is known that one has $\chi(F(r)) = 1$. By [10, Appendix II,Theorem 8.1]

$$\chi(X,E) = \chi(F(E),\pi^*E)\cdot\chi(F(r)) = \chi(F(E),\pi^*E)\,.$$

By the splitting principle, the bundle π^*E splits on $F(E)$

$$\pi^*E = L_1 \oplus \cdots \oplus L_r$$

into a sum of line bundles L_i. We then have

$$T(F(E),\pi^*E) = \sum_{i=1}^{r} T(F(E),L_i)$$

and ([10, Theorem 16.1.2])

$$\chi(F(E),\pi^*E) = \sum_{i=1}^{r} \chi(F(E),L_i)\,.$$

Observe that to conclude the proof, it suffices to show HRR for line bundles. Indeed, then we have

$$\chi(F(E),L_i) = T(F(E),L_i) \quad \text{for} \quad 1 \le i \le r\,,$$

and $\chi(X,E) = T(X,E)$. To achieve HRR for line bundles, we proceed as follows. By [10, Theorem 20.2.2], we have a fundamental equation for the arithmetic genus:

$$\chi(X) = \mathrm{td}(X)\,.$$

This is a consequence of three facts:

- $\chi(F(E)) = \chi(X)$ by [10, Theorem 20.2.1], hence $\chi(F(TX)) = \chi(X)$;
- $\mathrm{td}(F(E)) = \mathrm{td}(X)$ by [10, Theorem 14.3.1], hence $\mathrm{td}(F(TX)) = \mathrm{td}(X)$;
- $\chi(F(TX)) = \mathrm{td}(F(TX))$ due to [10, Theorem 20.1.1].

For a line bundle L on X we define

$$\chi(L)_X = \chi(X) - \chi(X, L^*)\,, \qquad T(L)_X = td(X) - T(X, L^*)$$

to be the *virtual characteristic* and *virtual genus* respectively. One can prove that for any line bundle L, the virtual characteristic and genus coincide: $\chi(L)_X = T(L)_X$ (see [10, Theorem 20.3.1]). A functional equation used in the proof also shows up at several other places in [10]: Theorem 11.3.1, Theorem 12.3.2, Theorem 17.3.1, Theorem 19.3.1 and Theorem 19.3.2. (We shall return to Theorem 11.3.1 in the identity (2).) Substituting L^* by L we obtain $T(X, L) = \chi(X, L)$, which completes the sketch of the proof.

The Hirzebruch–Riemann–Roch theorem was a starting point for several further results:

- Grothendieck's generalization of HRR for a proper morphism between algebraic varieties. This generalization is expressed by means of commutativity of a certain diagram involving homology groups and K-groups in algebraic geometry. This led to
- development of topological K-theory (Atiyah, Hirzebruch), which in turn served as an important instrument for
- the index theorem for elliptic operators (Atiyah, Singer).

Hirzebruch considered generalizations of $\chi(X, E)$. Introducing an (auxiliary) variable y, he defined the following χ_y*-characteristic*

$$\chi_y(X, E) = \sum_{p=0}^{n} \chi(X, E \otimes \wedge^p (TX)^*) y^p = \sum_{p=0}^{n} \sum_{q=0}^{n} (-1)^q\, h^{p,q}(X, E) y^p\,,$$

where $h^{p,q}(X, E) = \dim_{\mathbb{C}} H^q(X, E \otimes \wedge^p (TX)^*)$.

The Todd genus, virtual characteristic and other notions were generalized in a similar manner. In [10, Theorem 21.3.1] Hirzebruch gave a generalization of HRR for $\chi_y(X, E)$ (though he considered HRR, and not this latter generalization, as the central theorem in his book [10]).

In the article [14] in the volume *Algebraic Geometry: Hirzebruch 70* Alain Lascoux demonstrated that, applied to flag varieties, χ_y-characteristic becomes an adequate tool for studying algebra and combinatorics of the Macdonald polynomials.

Let $K_0(var/X)$ be the relative Grothendieck group of complex algebraic varieties over a variety X. In [6] Jean-Paul Brasselet, Jörg Schürmann and Shoji Yokura defined a natural transformation $T_{y*} : K_0(var/X) \to H_*(X) \otimes \mathbb{Q}[y]$ commuting with proper pushdown, which generalizes the corresponding Hirzebruch characteristic. It is a homology class version of the motivic measure corresponding to a suitable specialization of the well-known Hodge polynomial. This transformation unifies the Chern class transformation of MacPherson and Schwartz (for

$y=-1$), the Todd class transformation in the singular Riemann–Roch theorem of Baum–Fulton–MacPherson (for $y=0$) and the L-class transformation of Cappell-Shaneson (for $y=1$).

The Hirzebruch–Riemann–Roch theorem has found many applications. In our work with Vishwambhar Pati and Vasudevan Srinivas on the *diagonal property*, which says that on $X\times X$ there exists a bundle of rank $\dim X$ together with a section vanishing along the diagonal, HRR appeared to be very useful for studying the 3-dimensional quadric $X = Q_3$. Let $[Q_2], [L]$ and $[P]$ (the 2-dimensional quadric, line and point) be generators of the Chow groups $A^1(Q_3), A^2(Q_3)$ and $A^3(Q_3)$ respectively. The total Chern class of a vector bundle E on Q_3 has the form

$$1 + d_1(E)\,[Q_2] + d_2(E)\,[L] + d_3(E)\,[P]\,,$$

where $d_i(E) \in \mathbb{Z}$. It appears (see [21]) that to show absence of the diagonal property for Q_3 it is sufficient to prove that there is no bundle E of rank 3 over Q_3 with $d_3(E) = 1 = d_1(E)$. The latter follows from the fact that substituting these numbers into HRR

$$\chi(Q_3, E) = \frac{1}{6}(2d_1^3 - 3d_1 d_2 + 3d_2) + \frac{3}{2}(d_1^2 - d_2) + \frac{13}{6}d_1 + 3\,,$$

where $d_i = d_i(E)$, we obtain $\chi(Q_3, E) = \frac{15}{2} - 2d_2$, which is a contradiction.

In fact, the book [10] contains much more than a proof of HRR. In our work [15] with Adam Parusiński on the topological Euler characteristic of possibly singular complex hypersurfaces, at first we proved a formula for the Euler characteristic of

(from left) F. Hirzebruch, the author and A. Parusiński during *Hirzebruch 70* the zero locus of a section of a very ample line bundle. In order to generalize this formula to an arbitrary line bundle L, our strategy was to take a very ample line bundle M such that $\mathsf{L} \otimes \mathsf{M}$ is again very ample. But we were not able to make this idea work for several days...

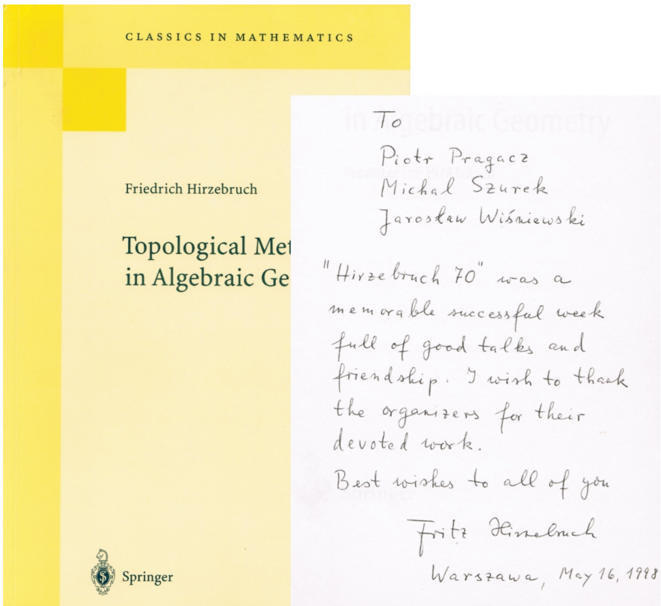
For a vector bundle E of rank r over a smooth complex manifold X let us define

$$\chi(\mathsf{X}|\mathsf{E}) = \int_{\mathsf{X}} c(\mathsf{E})^{-1}\, c_r(\mathsf{E})\, c(\mathsf{X})\,.$$

It is well known that this expression gives the (topological) Euler characteristic of the zero locus of a section which is transverse to the zero section of E. A few days after that, we found in [10] that there is an identity

$$2\chi(\mathsf{X}|\mathsf{L}\oplus\mathsf{M}) + \chi(\mathsf{X}|\mathsf{L}\otimes\mathsf{M}) = \chi(\mathsf{X}|\mathsf{L}) + \chi(\mathsf{X}|\mathsf{M}) + \chi(\mathsf{X}|\mathsf{L}\oplus\mathsf{M}\oplus\mathsf{L}\otimes\mathsf{M})\,. \qquad (2)$$

This is a very particular case of [10, Theorem 11.3.1], proved for Hirzebruch's virtual T_y-genus. With the help of this identity a proof of our formula for the Euler characteristic of a projective complex hypersurface went smoothly (see [15, p. 349]). Several years later we proved this formula for complete (compact) hypersurfaces – but this is a subject for another story...



As I write about the Hirzebruch–Riemann–Roch theorem, I am using the book *Topological methods in algebraic geometry* which was given by Professor Hirzebruch to me and my coorganizers, Michał Szurek and Jarosław Wiśniewski, as thanks for the organization of the conference *Hirzebruch 70* in Warsaw, in 1998.

The conference took place at the Banach Center residence on Mokotowska street. A lot of prominent algebraic geometers came to Warsaw on that occasion. The proceedings of that conference were published as the volume *Algebraic Geometry: Hirzebruch 70* of the AMS series *Contemporary Mathematics.*

Though one cannot see Michael Atiyah in this picture, he participated in the conference and gave a talk on the links between physics and algebraic geometry in the 1980s and 1990s, and also on the role of Friedrich Hirzebruch and his *Arbeitstagung* in the development of these links.

Since 2000 I have been running the IMPANGA algebraic geometry seminar at the Institute of Mathematics of the Polish Academy of Sciences (see the webpage [19], describing more broadly the IMPANGA algebraic geometry enviroment at this institute). Professor Hirzebruch was the first foreign speaker at the seminar. On May 7, 2001 he gave a talk *Old and new applications of characteristic classes.* The talk took place at his favorite place, i.e., at the Banach Center residence on Mokotowska street. The speaker received the traditional IMPANGA cup. A lot of advertisements of his talk were posted around over the institute; there was one in the elevator. In the evening I went to the National Opera with Fritz and his wife Inge. There, in the elevator, Fritz smiled: — In every second elevator in Warsaw there is an advertisement of my talk.

When I along with colleagues was organizing in 2010 the school and conference IMPANGA 10, Professor Hirzebruch agreed to be the Honourable Chairman of the Scientific Committee. Unfortunately, for a health reason he couldn't come in person to the Banach Center in Będlewo, but he took an active part in choosing conference speakers. Professor Hirzebruch's support was very important for the young participants of IMPANGA. In token of our gratitude, a speaker at IMPANGA 10, Masha Vlasenko, brought him a conference cup. This way there were now two IMPANGA cups at his office.

Earlier the participants of IMPANGA took an active part in the symposium *Manifolds in mathematics and in other fields* (IM PAN 2002) organized by Friedrich

Hirzebruch and Stanisław Janeczko. A large group of Polish algebraic geometers benefited from the hospitality of Professor Hirzebruch at the Max Planck Institute in Bonn: Grzegorz Banaszak, Andrzej Dąbrowski, Wojciech Gajda, Piotr Krasoń, Adrian Langer, Adam Parusiński, Tomasz Szemberg, Jarosław Wiśniewski, Jarosław Włodarczyk and the author. Those few years spent at MPIM were the most mathematically productive ones in my life. The cheerful atmosphere at the institute led by Professor Hirzebruch was steadily converted into advances at work. After the first year of my stay at MPIM he handed me his *Collected Papers* with an appropriate dedication and a humorous request: – May I please ask you to not just read my *Collected Papers* but also quote them; this way more mathematicians will get interested and the books will be selling better. The bibliography to [20] shows that I took his advice heartily.

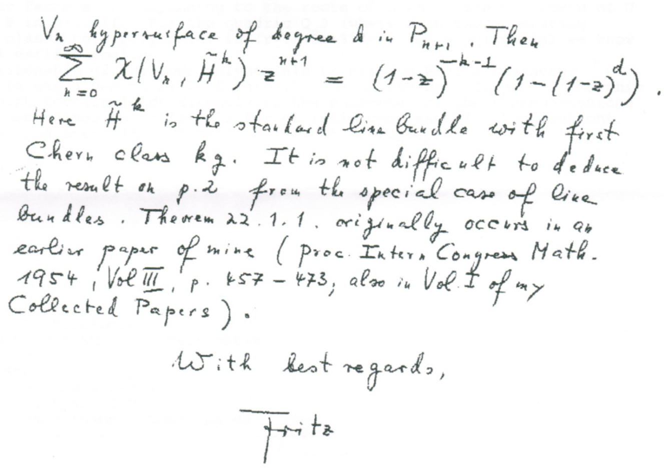
One of the topics I was working on in Bonn along with my doctoral student Jan Ratajski were Lagrangian, symplectic and orthogonal degeneracy loci. This is a beautiful part of Schubert calculus where geometry intertwines with algebra. We discovered an intriguing identity for the Schur functions in the variables x_i and their squares,

$$s_\lambda(x_1^2,\ldots,x_n^2)\, s_\rho(x_1,\ldots,x_n) = s_{\rho+2\lambda}(x_1,\ldots,x_n)\,, \tag{3}$$

where $\rho = (n, n-1, \ldots, 1)$ and $(\rho+2\lambda)_i = \rho_i + 2\lambda_i$.

This identity allowed us to prove a new Gysin formula for Lagrangian Grassmann bundles, but we kept feeling that we didn't understand it in sufficient depth. I confided to Professor Hirzebruch, and he suggested that we look at (3) from the point of view of quaternionic manifolds, pointing to several articles by himself and a paper by Peter Slodowy. And indeed with the use of quaternionic flag manifolds identity (3) becomes natural (see [20, (10.7)]).

As I already mentioned, HRR appeared very helpful to show that the 3-dimensional quadric does not have the diagonal property. Being asked, Professor Hirzebruch gave us a few pieces of advice on how the right-hand side of HRR can be effectively computed for complete intersections.



V_n hypersurface of degree d in P_{n+1}. Then

$$\sum_{k=0}^{\infty} \chi(V_n, \tilde H^k)\, z^{n+1} = (1-z)^{-k-1}\left(1-(1-z)^d\right).$$

Here $\tilde H^k$ is the standard line bundle with first Chern class kg. It is not difficult to deduce the result on p. 2 from the special case of line bundles. Theorem 22.1.1. originally occurs in an earlier paper of mine (Proc. Intern. Congress Math. 1954, Vol III, p. 457–473, also in Vol. I of my Collected Papers).

With best regards,

Fritz

Fritz cared a lot that the mathematicians hosted by the Max Planck Institute felt comfortable. They were often invited by him for lunch. When the institute was located in Beuel, he would invite his guests to the Italian restaurant *Tivoli*. When the institute moved to the center of Bonn, they would go to a small restaurant over the confectionery *Fassbender* located very close the new building of the institute. Fat Thursday is the culmination of the carnival at MPIM. This tradition was brought from the former building, because in Beuel the carnival procession takes place on this day. Being a guest of MPIM about the carnival time in 2001, I was amazed to find a necktie in my pigeonhole on Wednesday before Fat Thursday. I was told that there was a tradition to cut ties during the Thursday celebration. I also learned that it was Fritz that put the tie in my pigeonhole. The next day the tie was cut... — but not THAT ONE! The tie from Fritz remained a precious souvenir, and has been serving me ever since then.

Bonn has been a very attractive mathematical center. I was attending Professor Egbert Brieskorn's lectures at the university. These beautiful lectures (in German) highlighted relations of singularities and... art. In 2006 I had an opportunity to give two talks at Brieskorn's seminar on singularities. I started with expressing my gratitude for being able to speak at the seminar on singularity theory. Brieskorn replied instantly: – Pardon me, there is no singularity theory, there are only singularities. I benefited a lot from his numerous clever comments. Every Friday at 17:15 there was a colloquium at the university with very interesting talks. Professor Brieskorn told me that in order to encourage people to participate in the colloquium (at the initial stage) Hirzebruch used to say: — As Catholics weekly attend the mass, in a similar manner mathematicians should attend the colloquium.

Telling about his father, Professor Hirzebruch recalled that he was greatly respected by his pupils. As he was entering the class, it was sufficient for him to say: – Jungen! (– Boys!), and the class would fall silent.

Friedrich Hirzebruch was also gifted at working with people. When a problem arose, he would always manage the situation in a way that no conflict was possible. When he spoke on anything, he would concentrate on positive sides (though we knew there were negatives as well). He had a brilliant sense of humor. One could spot the following practice at the institute. Only one person would come to work in a suit and necktie, others were more casual: sweaters, ... Once that person — it was Fritz — was asked, why to torture oneself that much. His answer was: — If someone from outside were to visit our institute, they would notice without asking anyone, who is the director here!

At the institute we felt safe under Professor Hirzebruch's wing. We received enormous intellectual, spiritual and also financial support. He was continually advising, keeping our spirits up, helping.

Friedrich Hirzebruch was a fantastic mathematician. He solved the Riemann–Roch problem in all dimensions. His organizational skills made Bonn into an incredibly attractive mathematical center. A lot of Polish mathematicians, and particularly many IMPANGA participants, benefited and continue to benefit from it. He was open to everyone, whom he would help as much as he could.

In one of Don Zagier's texts on Friedrich Hirzebruch I found the words I would like to finish this article with:

> "*In many almost invisible ways, he made the people around him slightly better people, and the world around him a slightly better world.*"

Acknowledgements. My sincere thanks go to Masha Vlasenko for her translation and valuable comments. I am greatly indebted to Michael Atiyah for telling me about John A. Todd and his work on the arithmetic genus. I am also grateful to Grzegorz Banaszak, Gerard van der Geer, Adam Parusiński, Thomas Peternell and an anonymous referee for careful reading of the paper, and pointing out some corrections and improvements. I would like to thank Andrea Kohlhuber and Renata Podgórska for their help with pictures. Finally, I wish to thank Brian Harbourne for his linguistic help.

References

[1] M. Atiyah, *Friedrich Hirzebruch – an appreciation*, Proceedings of the Hirzebruch 65 Conference in Algebraic Geometry (Ramat Gan, 1993) (M. Teicher, ed.), Israel Math. Conf. Proc. 9 (1996), 1–5.

[2] M. Atiyah, *Friedrich Ernst Peter Hirzebruch, 17 October 1927 – 27 May 2012*, Biogr. Mem. Fellows R. Soc. 60 (2014), 229–247.

[3] M. Atiyah, C. Bär, J.-P. Bourguignon, G.-M. Greuel, Y. I. Manin, M. Sanz-Solé, *Friedrich Hirzebruch Memorial Session at the 6th European Congress of Mathematics (Kraków, July 5th, 2012)*, Eur. Math. Soc. News 85 (September 2012), 12–20.

[4] M. Atiyah, D. Zagier (coordinating editors), *Friedrich Hirzebruch (1927–2012)*, Notices Amer. Math. Soc. 61 (2014), 2–23.

[5] J.-P. Bourguignon, *Questions au Professeur Hirzebruch*, Gazette de Mathematiciens 53 (Juin 1992), 11–19.

[6] J.-P. Brasselet, J. Schürmann, S. Yokura, *Hirzebruch classes and motivic Chern classes for singular spaces*, J. Topol. Anal. 2 (2010), 1–55.

[7] E. Brieskorn, *Singularities in the work of Friedrich Hirzebruch*, Surv. Differ. Geom. 7 (2007), 17–60.

[8] É. Ghys, A. Ranicki, *Signatures in algebra, topology and dynamics*, arXiv:1512.09258.

[9] H. Grauert, G. Harder, R. Remmert, *Curriculum vitae mathematicae*, Math. Ann. 278 (1987), iii–viii.

[10] F. Hirzebruch, *Topological Methods in Algebraic Geometry*, 3rd edition, Springer 1966.

[11] F. Hirzebruch, *Prospects in Mathematics. The signature theorem: reminiscences and recreation*, Ann. Math. Stud. 70 (1971), 3–31.

[12] F. Hirzebruch, *Why do I like Chern, and why do I like Chern classes?*, Notices Amer. Math. Soc. 58 (2011), 1231–1234.

[13] M. Kreck, *Video interview with Friedrich Hirzebruch*, Simons Foundation website (2011).

[14] A. Lascoux, *About the "y" in the* χ_y*-characteristic of Hirzebruch*, in Algebraic Geometry: Hirzebruch 70, P. Pragacz, M. Szurek and J. Wiśniewski (eds.), Contemp. Math. 241, Amer. Math. Soc., Providence RI, 1999, 285–296.

[15] A. Parusiński, P. Pragacz, *A formula for the Euler characteristic of singular hypersurfaces*, J. Alg. Geom. 4 (1995), 337–351.

[16] P. Pragacz, *Notes on the life and work of Alexander Grothendieck*, in Topics in Cohomological Studies of Algebraic Varieties, Trends in Mathematics, Birkhäuser, Basel 2004, xi–xxviii.

[17] P. Pragacz, *Notes on the life and work of Józef Maria Hoene-Wroński*, in Algebraic Cycles, Sheaves, Shtukas and Moduli, Trends in Mathematics, Birkhäuser, Basel 2007, 1–20.

[18] P. Pragacz, *Friedrich Hirzebruch – garść reminiscencji*, Wiadomości Matematyczne 52(1) (2016), 37–51.

[19] `https://www.impan.pl/~pragacz/impanga.htm` (accessed: 07.02.2017).

[20] P. Pragacz, J. Ratajski, *Formulas for Lagrangian and orthogonal degeneracy loci;* $\widetilde{Q}$*-polynomial approach*, Compositio Math. 107 (1997), 11–87.

[21] P. Pragacz, V. Srinivas, V. Pati, *Diagonal subschemes and vector bundles*, Pure Appl. Math. Quarterly 4(4), part 1 of 2, Special Issue in honor of Jean-Pierre Serre (2008), 1233–1278.

[22] D. Zagier, *Life and work of Friedrich Hirzebruch*, Jahresber. Dtsch. Math.-Ver. 117 (2015), 93–132.

Piotr Pragacz, Institute of Mathematics, Polish Academy of Sciences, Śniadeckich 8, 00-656 Warszawa, Poland

E-mail: P.Pragacz@impan.pl