Examples to even orthogonal Pieri

Piotr Pragacz and Jan Ratajski

This manuscript contains a collection of examples that may be helpful in reading our paper: **A Pieri-type formula for even orthogonal Grassmannians, Fund. Math. 178 (2003), 49-96**.

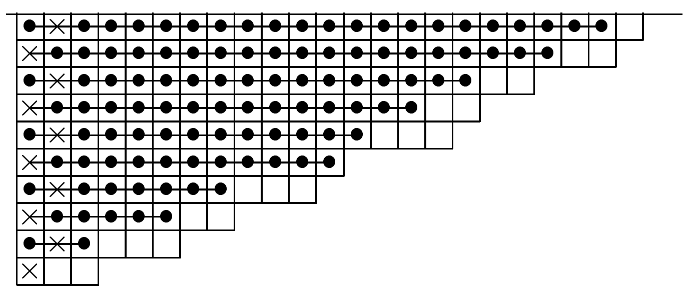
After these examples, we give an errata to that paper.

Finally, on page 10, an errata to our former (related) paper is added.

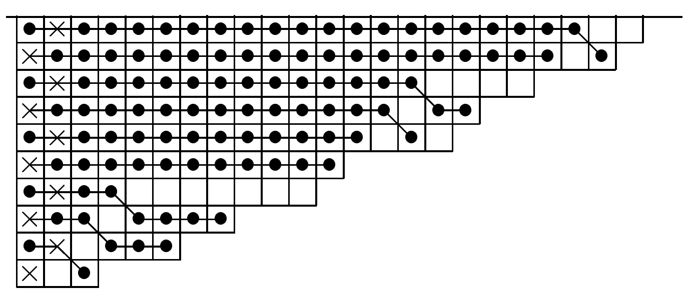
Example 1 $\lambda = (22, 20, 17, 15, 13, 12, 8, 6, 3)$,

$\mu = (23, 22, 19, 17, 16, 12, 11, 8, 6, 3)$.

The maximal deformation changes

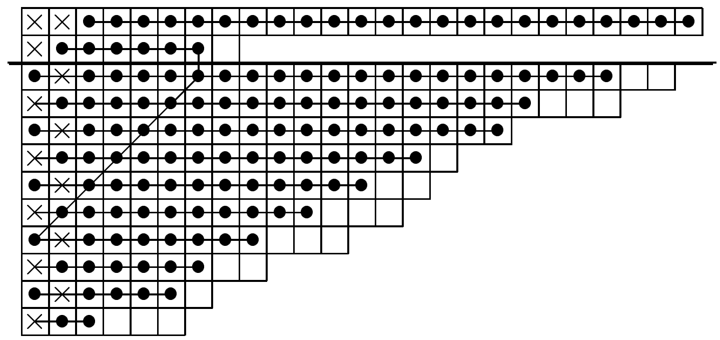


to

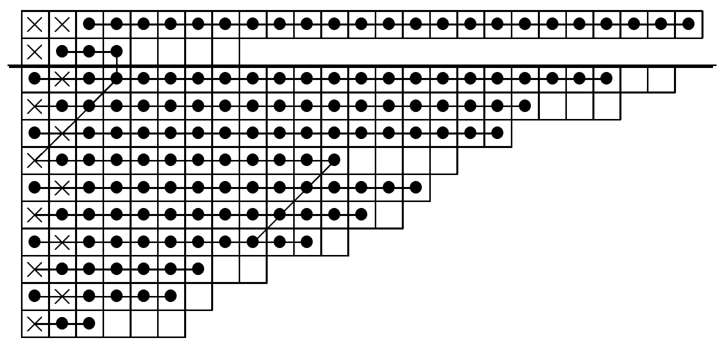


Here is an example with several e-transformations:

Example 2 $\lambda = (25, 7//22, 19, 18, 15, 13, 11, 9, 7, 6, 3)$,
$\mu = (25, 8//24, 22, 18, 16, 15, 14, 12, 9, 7, 6)$.
Three e-transformations change



to



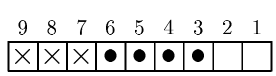
(Here $m \geq 25$. In all later examples one should take m large enough.)

Here are some examples illustrating how do act on generating functions operators composed of divided differences and simple reflections.

Example 3 *We apply the operators of boxes from left to right to* $E_{\mathbf{a}}$ *and obtain* $(scalar)E_{\mathbf{a}'}$*;* $\bullet$ *denotes a D-box and empty boxes are* $\sim$*D-boxes.*

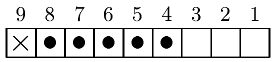
$$m = 9 \qquad\qquad \mathbf{a} = (a_9, a_8, a_7, a_6, a_5, a_4, a_3, a_2, a_1)$$

1. *Action of the operators associated with a row in* D^t_μ*:*

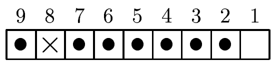


$$\mathbf{a}' = (a_9, a_8, a_6, a_5, a_4, a_3, 0, 0, 0)$$

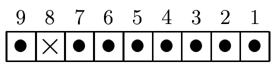
2. *Action of the operators associated with a row in D^b_μ:*



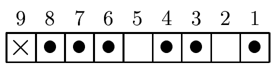
$$\mathbf{a}' = (a_8, a_7, a_6, a_5, a_4, 0, 0, 0, 0)$$



$$\mathbf{a}' = (-a_8, a_7, a_6, a_5, a_4, a_3, a_2, 0, 0)$$



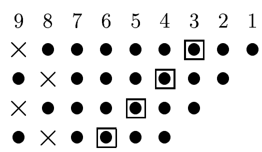
$$\mathbf{a}' = (-a_8, a_7, a_6, a_5, a_4, a_3, a_2, a_1, -a_9)$$

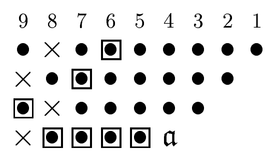


$$\mathbf{a}' = (a_8, a_7, a_6, 0, a_4, a_3, 0, a_1, 0)$$

3. *Action of the operators associated with a sequence of consecutive rows in D^b_μ containing only D-boxes.*

$\mathbf{a} = (*, *, *, *, *, *, b, *, *)$ $\quad$ $\mathbf{a} = (*, *, *, b, *, *, *, *, *)$

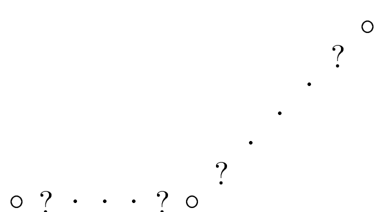


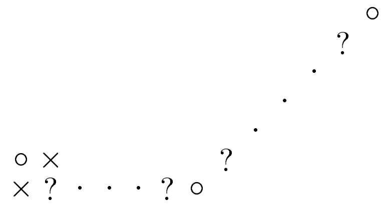


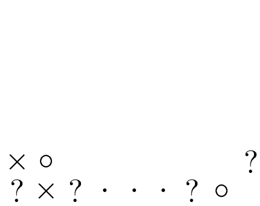
$\mathbf{a}' = (*, *, b, *, *, *, *, *, *)$ $\quad$ $\mathbf{a}' = (*, *, *, *, -b, *, *, *, *)$

The symbol ⊡ denotes the operators which move b; in the second picture, $\mathbf{a}'$ is obtained after applying all operators of boxes preceding $\mathfrak{a}$.

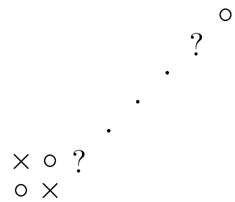
Here are some examples of the configurations of three $\sim D$-boxes in the bottom part that give vanishing:







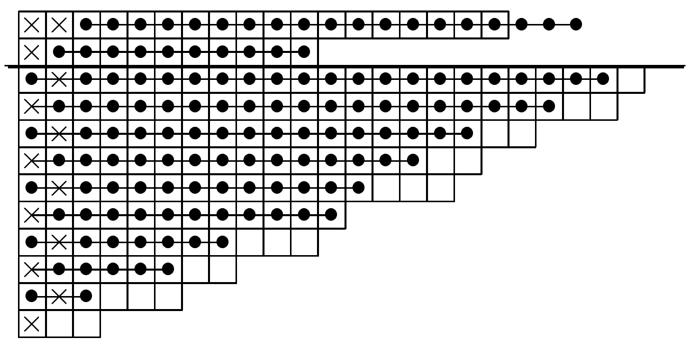
also:



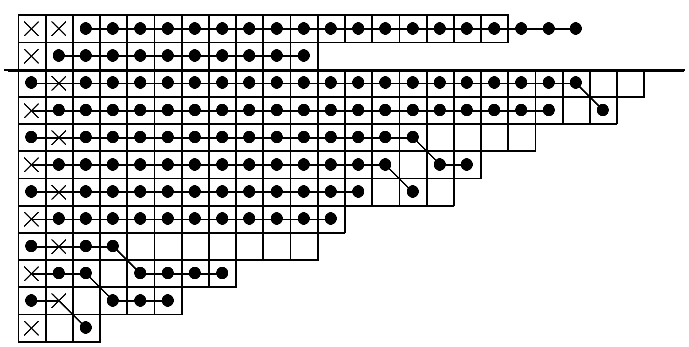
(Above, "?" can be $\circ$, $\bullet$, or $\times$; and the skew directions are all antidiagonal.)

We now pass to examples illustrating the recipies for finding the unique D such that $\partial_\mu^D(E) \neq 0$.

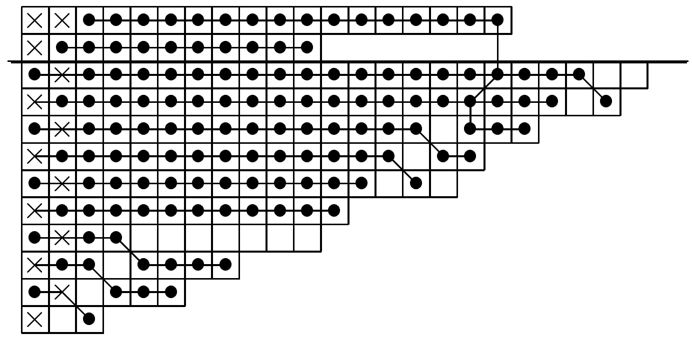
Example 4 $\lambda = (21, 11//22, 20, 17, 15, 13, 12, 8, 6, 3)$ *(type 1),*
$\mu = (18, 11//23, 22, 19, 17, 16, 12, 11, 8, 6, 3)$ *(type 1).*



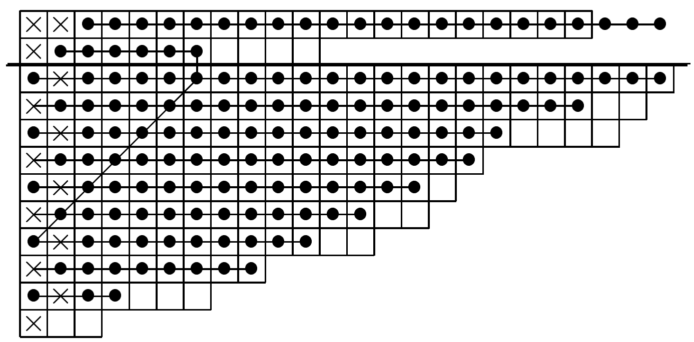
We perform the maximal deformation of D^b_λ *in* D^b_μ*:*



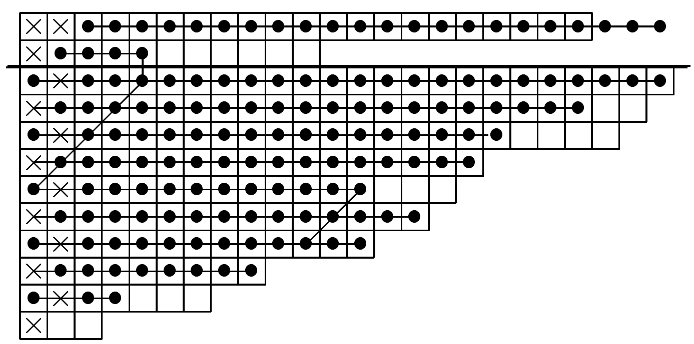
The final deformation of the v-ribbons (here only the v_β-ribbon is deformed) looks like:



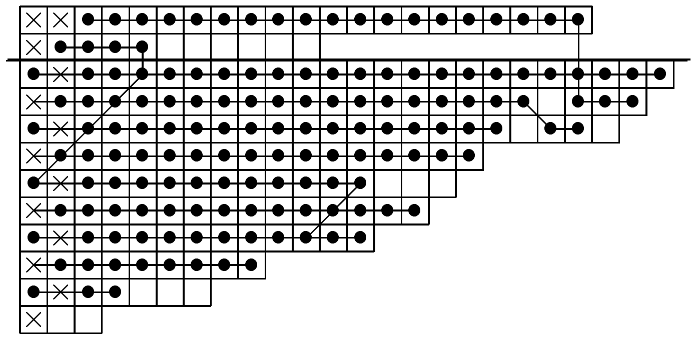
Example 5 $\lambda = (24, 7//24, 21, 18, 17, 15, 13, 11, 9, 4)$ *(type 2)*, $\mu = (21, 11//24, 23, 22, 17, 16, 15, 13, 9, 7, 3)$ *(type 1)*.



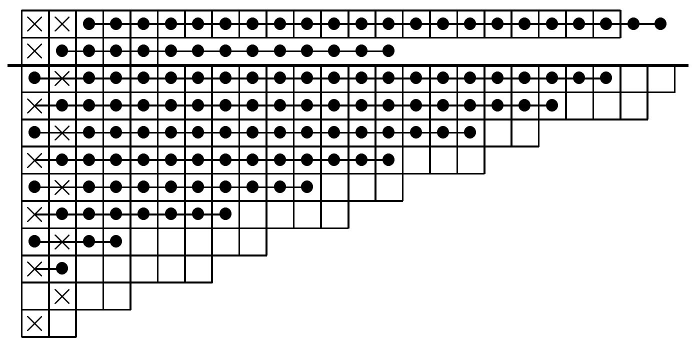
We perform twice the e-transformations:



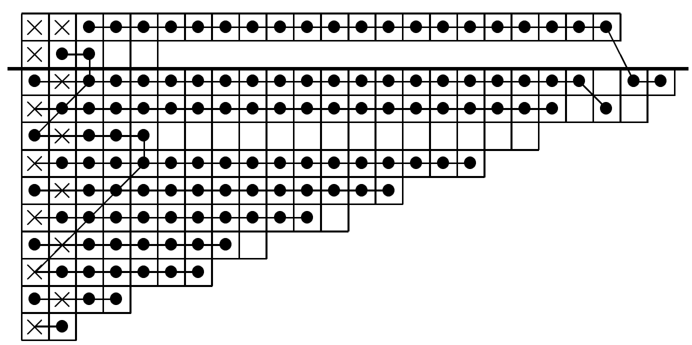
We deform the z-ribbons and the v_β-ribbon from D^t:



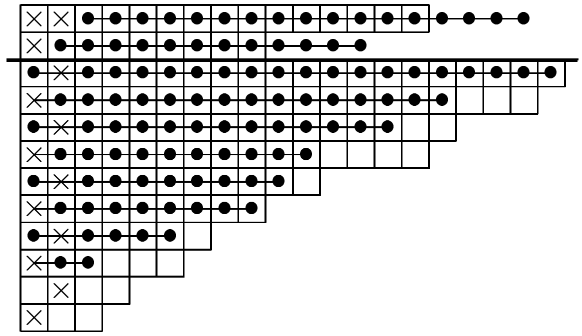
Example 6 $\lambda = (24, 14//22, 20, 17, 14, 11, 8, 4, 2)$ *(type 1),*
$\mu = (22, 5//24, 23, 19, 17, 14, 13, 10, 8, 4, 2)$ *(type 2).*



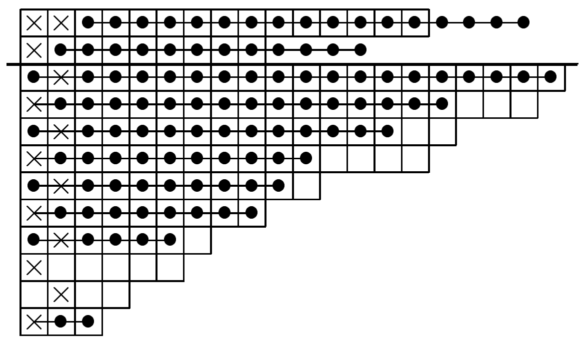
The resulting deformed diagram is:



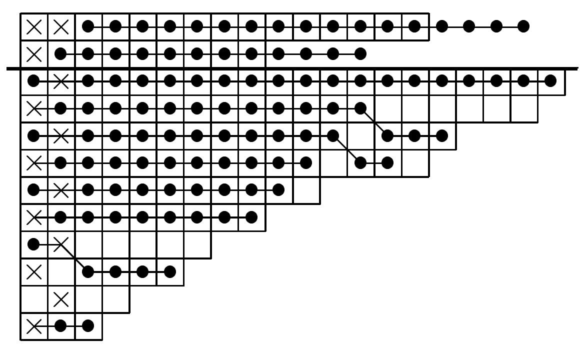
Example 7 $\lambda = (19, 13//20, 16, 14, 11, 10, 9, 6, 3)$ *(type 1)*,
$\mu = (23, 9//20, 19, 16, 15, 11, 9, 7, 6, 4, 3)$ *(type 2)*.



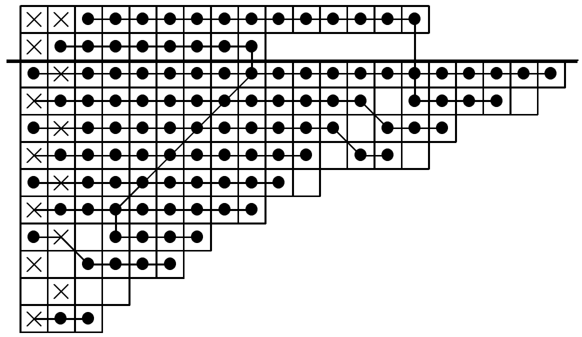
We perform the push down operation:



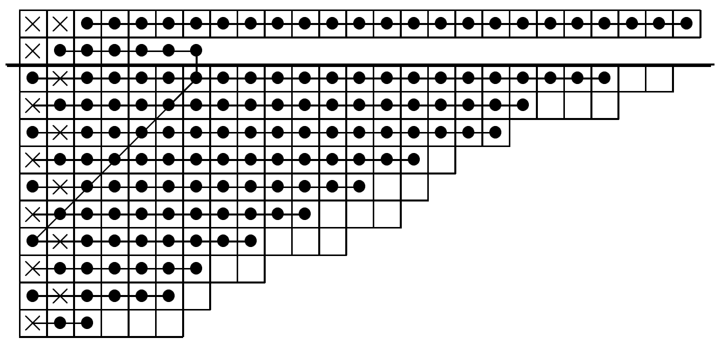
We break the ribbons:



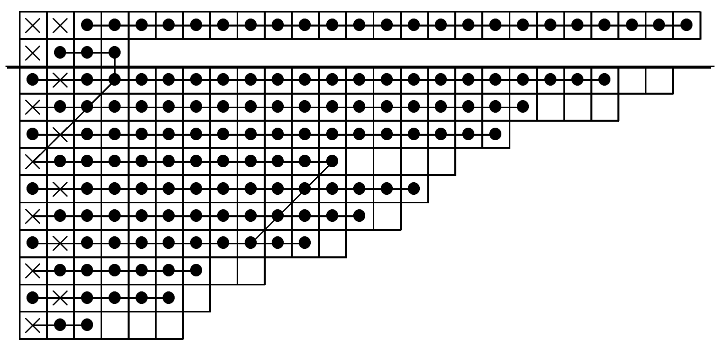
We deform the v_α-ribbon from D^t (and here also the v_β-ribbon):



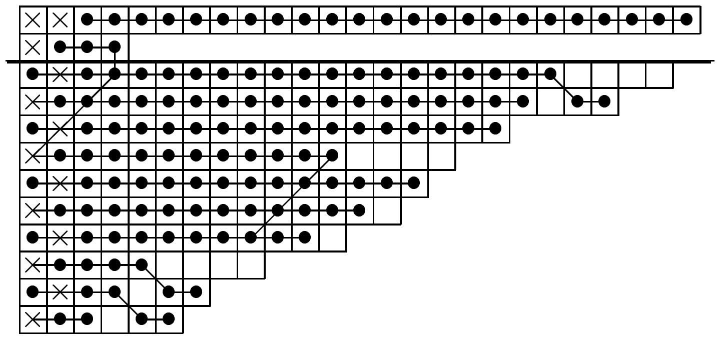
Example 8 $\lambda = (25, 7//22, 19, 18, 15, 13, 11, 9, 7, 6, 3)$ *(type 2),*
$\mu = (25, 4//24, 22, 18, 16, 15, 14, 12, 9, 7, 6)$ *(type 2).*



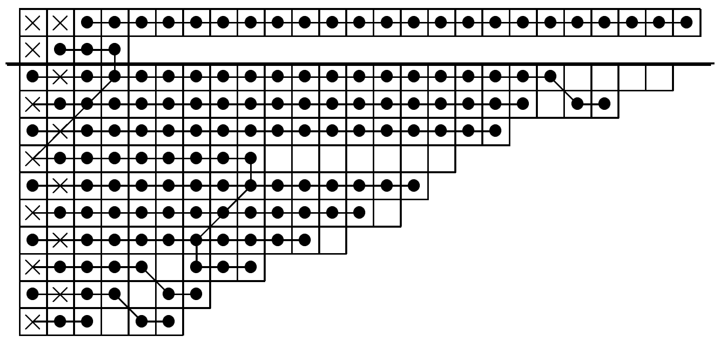
We apply three e-transformations:



We apply the maximal deformation:



We deform the v_α-ribbon:



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Errata to: P. Pragacz, J. Ratajski, A Pieri-type formula for even orthogonal Grassmannians, Fund. Math. 178 (2003), 49-96.

It should read:

p.54^3 " ... $= \lambda_\alpha^t + \lambda_s^b$,"

p.57, line 11 under the picture: " ... the μ_α^tth element ... "

p.58, line 3 over the picture: " ... the word obtained by ... "

p.63_{13} " ... below. With variables $x_1, \ldots, x_m$ replacing $\varepsilon_1, \ldots, \varepsilon_m$, let "

p.64^5 " form a regular sequence. The ... "

* * * * *

Errata to: P. Pragacz, J. Ratajski: “A Pieri-type theorem for Lagrangian and odd orthogonal Grassmannians”, J. Reine Angew. Math. 476 (1996), 143–189.

It should read:

p.147^{11} “ ... and (d_r) increases more slowly than (v_r) ...”

p.147^{13} “ $v_{m-n} \leq$ ”

p.149^{9} “ variables replacing $e_1, \ldots, e_m$.) By [D1] ... ”

p.149_{5} “ $+(-1)^{i-1} h_i($ ”

p.150^{3} “ $\sum_{k+l=i} e_k($ ”

p.150^{4} “ $\sum_{k+l=i} (-1)^l e_k($ ”

p.$151^{4,5}$ “$P(T) = \ldots$ ”

p.156_{15} “ $\ldots, a_{i+1}, a_{i-1}, \ldots$ ”

p.157_{11} “ ... class of the dual of the tautological ... ”

p.158_{10} “ $\ldots \circ \partial_{11} \circ s_{12} \circ \partial_{13} \circ s_{14} \circ \partial_5 \circ \ldots$ ”

p.158_{12} “ $\ldots \circ \partial_3 \circ s_4 \circ \ldots$ ”

p.168_{8} “ ... such that D^t has rows of decreasing lengths ”

p.168_{3} “ ... in $D^t_\mu \setminus D^t$, ”

p.179^{1} “ ... of boxes ... ”

p.180^{7} “ the row ...”

p.186^{7} “ $\Delta_{\mathfrak{a}}(E)$ ”

p.187_{10} “... for the non-extremal component ...”

Corollary 1.7 follows from the isomorphism displayed before Lemma 1.5 because $e_i(x_1^2, \ldots, x_m^2)$, $1 \leq i \leq m$, form a regular sequence.

In the picture on p.161, each “$, 3$” is to be changed to “$, 2$” and each “$, 8$” is to be changed to “$, 3$”.

Of course, the picture on p.179 is reproduced upside down.

In the lowest picture on p.181, the length of the lowest pure v-ribbon should be 3 (and not 2).

In the second picture in the last row on p.182, in the first bottom row should be: “ $\cdot\ \cdot\ \circ\ \circ\ \circ$ ”.