Alain Lascoux (1944-2013)

Alain Lascoux was one of the most active participants of the conference on Schubert calculus in Osaka in July 2012. There, I met him for the last time (later, we exchanged a couple of e-mails). During the conference in Osaka, he gave a talk on tableaux and Eulerian properties of the symmetric group, making use of Ehresmann-Bruhat order and keys with applications to Demazure modules and postulation of Schubert subvarieties in flag manifolds. He posed many questions during the lectures of the conference, and interacted a lot with participants, and especially with graduate students. He passed away in Paris on October 20, 2013.

Much of Alain Lascoux's research is devoted to Schubert calculus or inspired by it. Lascoux's work concerns mainly (algebraic) combinatorics, but also algebraic geometry, representation theory and commutative and noncommutative algebra. In fact, he came to combinatorics from algebraic geometry, and the intuitions from the latter domain (and especially from Schubert calculus) were always present in his work. His results were often placed on the borders of the aforementioned domains.

His papers on Schubert calculus concern: the coefficients of intersections of Schubert varieties, determinants in Chern classes, Schubert classes, i.e., the cohomology classes of Schubert subvarieties in Grassmannians and flag manifolds, Grassmannian extensions of λ-rings, Chern classes of tensor products of vector bundles, classes of degeneracy loci, the degree of the dual variety to a Grassmannian, Chern classes of flag varieties, the Littlewood-Richardson rule, the spaces of complete correlations and quadrics, Grothendieck rings of flag varieties, Thom polynomials and others.

Lascoux worked for 20 years with Marcel-Paul Schützenberger on properties of the symmetric group. They wrote many articles together, and had a major impact on the development of algebraic combinatorics. They succeeded in enhancing the combinatorial understanding of various algebraic and geometric questions in representation theory.

The favourite objects and principal tools in Lascoux's mathematical work were: tableaux (especially Young tableaux), symmetric functions (especially Schur functions but also noncommutative symmetric functions), determinants, polynomials (see below), operators on polynomial rings (especially various variants of divided differences), reproducing kernels, Hecke algebras, λ-rings of Grothendieck, flag varieties and Schubert varieties.

In algebraic geometry, he worked in enumerative geometry, enumerative theory of singularities and on singular loci of Schubert varieties. In representation theory, he studied symmetric and full linear groups, Young idempotents, Hecke algebras, crystal graphs, q-analogues of weight multiplicities, Kostka-Foulkes polynomials, Macdonald polynomials and Kazhdan-Lusztig polynomials. In combinatorics, his most famous contributions are related to tableaux and plactic monoid (with Schützenberger).

The Cauchy formula was the favourite one for Lascoux: because of its simplicity, profoundness and many incarnations.

In the early 1980's, Lascoux discovered with Schützenberger that the classical Schur functions are very particular cases of Schubert polynomials, defined by them via Newton's divided differences. They were polynomial lifts of the Schubert classes for flag manifolds. Even more natural were double Schubert polynomials, which received later a transparent geometric interpretation as the cohomology classes of degeneracy loci of morphisms between vector bundles from two flags (a result of Fulton).

Operators were often present in Lascoux's work. Apart from divided differences, used in the theory of Schubert polynomials and their variations (e.g. isobaric divided differences in the theory of Grothendieck polynomials), he also used vertex operators. I know that in the last years of his life, Lascoux worked on his monograph "Polynomials", where all these ideas and results about operators and polynomials are developed. As far as I know, the book just requires a finishing touch.

Let me finish with some personal reminiscences. I met Alain for the first time in Spring 1978 during his first visit to Poland. He taught us the syzygies of determinantal varieties, using the Bott theorem on cohomology of homogeneous bundles and Schur functors. In November and December 1978, I visited him in Paris, where he taught me Schubert calculus and symmetric functions. Then, I visited him in Paris many times (as he used to say: “trente six fois”). He was my principal teacher in mathematics. Between 1980 and 2010, we wrote together 15 papers, and most of them were devoted to or inspired by Schubert calculus. Also, many other of my papers are inspired by his papers.

Alain Lascoux was a master of Schubert calculus. He knew its classical sources, and had a deep understanding of its importance and applications to contemporary mathematics.

Piotr Pragacz, December 2014