Conditions on renormalizable maps yielding small moduli.

Since Sullivan's work on Feigenbaum's universality, complex bounds for polynomial-like renormalizations play a key role in the polynomial dynamics. On the other hand, examples by Douady and Hubbard show that the bounds do not hold in general. Until now, these have been the only known examples of such kind. We show that such bounds do not exist in many other maps, quite different from Douady-Hubbard's ones.

To this end we prove an explicit upper bound for the modulus of renormalization of a rational function and derive several corollaries. E.g., if an infinitely renormalizable map admits a sequence of satellite renormalizations with relative periods tending to infinity and moduli of annuli bounded away from 0, then moduli of annuli on the next level converge to 0. As another application, we give an explicit requirement of a fast convergence of relative periods to infinity which yields vanishing moduli. Under this condition, the MLC holds if the map is a quadratic polynomial. We also give conditions for vanishing moduli for a sequence of non-satellite renormalizable maps and provide examples showing that the conditions are optimal.

Joint work with Alexander Blokh, Lex Oversteegen and Vladlen Timorin.