Discrete and locally compact Cantor orbits in topologically transitive cylinder transformations

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Let X be a compact metric space and $T : X \longrightarrow X$ be a homeomorphism of X. Let $f : X \longrightarrow \mathbb{R}$ be a continuous function (called *a cocycle*). By a *cylinder transformation* we mean a homeomorphism. $T_f : X \times \mathbb{R} \longrightarrow X \times \mathbb{R}$ (or rather a $\mathbb{Z}$–action generated by it) given by the formula

$$T_f(x, r) = (Tx, f(x) + r).$$

H. Poincaré addressed the problem of what types of orbits may coexist in such a system ([5]). If X is a monothetic group then the following trichotomy takes place: 1. either f has nonzero mean (equivalently all orbits are discrete) or 2. f is a coboundary (equivalently every orbit clouser is homeomorphic to X) or 3. T_f is topologically transitive (has a dense orbit) ([2], [4]). Therefore 3. is the only interesting case. Since a cylindrical transformation cannot be itself minimal ([1]), it has to have non dense orbits provided its top. trans..

In [3] we delivered a method to construct a few classes of top. trans. cylinder transformations with a relatively large set of discrete orbits of various types (with respect to their boundedness). Using this method we show how to construct top. trans. cylindrical transformations possessing orbits of quite complicated nature. Namely ω-limit sets of some points are unions (countable or not) of locally compact Cantor sets.

References

[1] A.S. Besicovitch, *A problem on topological transformations of the plane. II*, Proc. Cambridge Philos. Soc. **47**, (1951), 38-45.

[2] W.H. Gottschalk, G.A. Hedlund, *Topological Dynamics*, AMS Colloquium Publications, **36** (1955).

[3] J. Kwiatkowski, A. Siemaszko, *Discrete orbits in topologically transitive cylindrical transformations.* Discrete Contin. Dyn. Syst. **27** (2010), no. 3, 945961.

[4] M. Lemańczyk, M.K. Mentzen, *Topological ergodicity of real cocycles over minimal rotations*, Monatsh. Math., **134** (2002), 227-246.

[5] H. Poincaré, *Mémoire sur les courbes définies par une équation différentielle*, 1882.