Visibility for self-similar sets of dimension one in the plane

by

B. Solomyak, K. Simon

For $a \in \mathbb{R}^2$, let P_a be the radial projection from a:

$$P_a : \mathbb{R}^2 \setminus \{a\} \longrightarrow S^1, \quad P_a(x) = \frac{(x-a)}{|x-a|}.$$

If a Borel set $\Lambda \subset \mathbb{R}^2$ is "big" (has Hausdorff dimension greater than one)

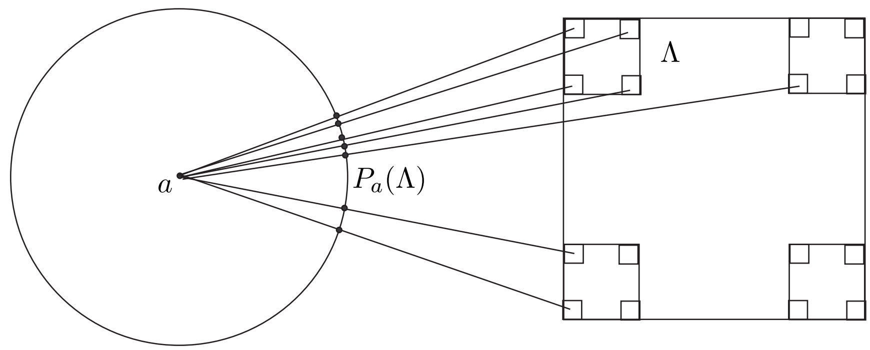

FIGURE 1. The radial projection of the four corner set

then the one dimensional Lebesgue measure of $P_a(\Lambda)$ is positive. In this case we say that the set is visible from the point $a \in \mathbb{R}^2$. If the set Λ is "small" (has Hausdorff dimension less than one) then $P_a(\Lambda)$ has zero one dimensional Lebesgue measure and we say that Λ is invisible from the point a. We discuss the problem of visibility of $\Lambda \subset \mathbb{R}^2$ from a point $a \in \mathbb{R}^2$ in the case when Λ is neither small nor big. More precisely, when Λ is a self-similar set of Hausdorff dimension one satisfying the open set condition.