Book of abstracts

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I. Minicourse

Paul Tod University of Oxford

Conformal methods in General Relativity with application to Conformal Cyclic Cosmology

Introduction, conventions and basic theory: 2 lectures

- Differential geometric conventions.
- Conformal rescaling, conformal weight, conformal geodesics.
- Important examples: de Sitter, anti-de-Sitter, Minkowski.
- Asymptotic simplicity and consequences.
- Behaviour of matter fields under rescaling: Maxwell fields, conformal scalars, Vlasov and Boltzmann.

Cosmology: 1 lecture

- FRW and conformal rescaling (at both ends).
- The conformal Einstein equations: data at infinity and data at the Bang; examples.

Conformal Cyclic Cosmology: 2 lectures

- The Weyl curvature hypothesis.
- Conformal gauge singularities: initial value problems for perfect fluid, Einstein-Vlasov and other matter models.
- The 'outrageous suggestion'.
- The equations of CCC. The VBE and fading rest-mass.
- Physical consequences, particularly 'circles in the sky'.

II. Invited talks

Luis J. Alías Universidad de Murcia

Trapped submanifolds in de Sitter space

The concept of trapped surfaces was originally formulated by Penrose for the case of 2-dimensional spacelike surfaces in 4-dimensional spacetimes in terms of the signs or the vanishing of the so-called *null expansions*. This is obviously related to the causal orientation of the mean curvature vector of the surface, which provides a better and powerful characterization of the trapped surfaces and allows the generalization of this concept to codimension two spacelike submanifolds of arbitrary dimension n. In this sense, an n-dimensional spacelike submanifold Σ of an $(n+2)$-dimensional spacetime is said to be *future trapped* if its mean curvature vector field $\mathbf{H}$ is timelike and future-pointing everywhere on Σ, and similarly for *past trapped*. If $\mathbf{H}$ is lightlike (or null) and future-pointing everywhere on Σ then the submanifold is said to be *marginally future trapped*, and similarly for *marginally past trapped*. Finally, if $\mathbf{H}$ is causal and future-pointing everywhere, the submanifold is said to be *weakly future trapped*, and similarly for *weakly past trapped*. The extreme case $\mathbf{H} = \mathbf{0}$ on Σ corresponds to a *minimal* submanifold.

In this lecture we consider codimension two trapped submanifolds contained into a null hypersurface of de Sitter space. In particular, for the case of compact submanifolds into the light cone of de Sitter space, we show that they are conformally diffeomorphic to the round sphere. This fact enables us to deduce that the problem of characterizing compact marginally trapped submanifolds into the light cone is equivalent to solving the Yamabe problem on the round sphere, allowing us to obtain our main classification result for such submanifolds. We also fully describe the codimension two compact marginally trapped submanifolds contained into the past infinite of the steady state space and characterize those having parallel mean curvature field. Finally, we consider the more general case of codimension two complete, non-compact, weakly trapped spacelike submanifolds contained into the light cone.

This is part of our recent joint work with Verónica L. Cánovas (from Murcia) and Marco Rigoli (from Milano), which can be found in the paper *Trapped submanifolds contained into a null hypersurface of de Sitter spacetime*, to appear in Commun. Contemp. Math. (`https://doi.org/10.1142/S0219199717500596`).

Miguel Brozos-Vázquez Universidade da Coruña

Four-dimensional quasi-Einstein manifolds

A quasi-Einstein manifold is a quadruple (M, g, f, μ), where (M, g) is a pseudo-Riemannian manifold, f is a smooth function on M and $\mu \in \mathbb{R}$, such that the tensor $\mathrm{Hes}_f + \rho - \mu df \otimes df$ is a multiple of the metric, i.e. the quasi-Einstein equation is satisfied:

$$\mathrm{Hes}_f + \rho - \mu df \otimes df = \lambda\, g.$$

Hes_f denotes the Hessian of f, ρ denotes de Ricci tensor and λ is a function on M. We will distinguish the case in which λ is a constant and treat it separately.

Quasi-Einstein manifolds generalize several well-known families of manifolds, such as *Einstein* manifolds, *gradient Ricci solitons*, or *κ-Einstein solitons* among others.

It is a hard task to give a classification of quasi-Einstein manifolds in general. Since the equation provides information directly on the curvature through the Ricci tensor, it is common to impose natural restrictions on the conformal Weyl tensor and provide classification results in a more specific setting. We will consider different conditions in this line.

There are two situations genuinely different depending on the character of ∇f. If $g(\nabla f, \nabla f) \neq 0$, the level hypersurfaces of f are non-degenerate. This is what happens if the metric is definite. However, if $g(\nabla f, \nabla f) = 0$, the level hypersurfaces of f are degenerate and new examples show up in indefinite signature. In this talk we will focus on Lorentzian and neutral signature manifolds.

Erasmo Caponio Politecnico di Bari

On a Finslerian generalisation of standard static and stationary spacetimes

Recently, Finsler spacetimes have attracted a renewed interest only partially justified by their applications to theories of gravity or to effective field theories with Lorentz violation. Rather, they widely generalise the notion of Lorentzian spacetime but retain several familiar features of it. I will try to focus on some aspects of this generalisation, referring in particular to spacetimes with a timelike Killing vector field.

Piotr T. Chruściel Universität Wien

The mass of asymptotically hyperbolic manifolds

I will review the notion of mass for asymptotically hyperbolic manifolds, and present some new and old positivity results.

Wolfgang Globke Universität Wien

Isometry Lie algebras of indefinite homogeneous spaces of finite volume

An interesting problem in pseudo-Riemannian geometry is the study of manifolds with an indefinite metric that have finite volume and are homogeneous for the action of its isometry group. So far, these have been studied only in the case of Lorentzian metrics independently by Adams & Stuck and by Zeghib. Here, we develop a structure theory for the Lie algebras of groups that can act isometrically and effectively on such a manifold with more general metric index. Our first main result is a strong invariance property for the pseudo-scalar products induced on the Lie algebras by the indefinite metric. For metric index up to two, we can then give a classification of these Lie algebras. Examples show that for metric index three or higher the structure becomes significantly more complicated.

this talk is based on joint work with Oliver Baues and Abdelghani Zeghib (arXiv:1803.10436).

Philippe G. LeFloch Sorbonne Université, Paris

Euclidian-Hyperboloidal foliations and CMC-harmonic coordinates

I will present two construction methods for foliating Einstein spacetimes. On one hand, for perturbations of Minkowski spacetime, I have introduced (jointly with Yue Ma, Xi'an Jiatong) the Euclidian-Hyperboloidal Method, as we call it. This construction is based on a (3+1)-foliation of the interior of a light cone by hyperboloids, combined with a foliation by asymptotically Euclidian hypersurfaces. This method has allowed us to establish the first result of global nonlinear stability of Minkowski space for massive fields.

On the other hand, I will present the construction (obtained with B.-L. Chen, Guangzhou) of a local canonical foliation by CMC hypersurfaces, combined with spatially harmonic coordinates. This construction solely involves assumptions on the curvature and injectivity radius in a local domain of the spacetime.

Jerzy Kijowski Center for Theoretical Physics Polish Academy of Sciences

Trautman-Bondi mass: How much energy is carried by gravitational waves

In his Ph.D. thesis (1958) Andrzej Trautman has shown how to calculate amount of energy carried by gravitational waves. This approach was later simplified by Roger Penrose's definition of "null infinity". In my talk I show that these phenomena are universal and occur not only in General Relativity. In particular, they arise in any special-relativistic field theory (e.g. in – linear or nonlinear – electrodynamics). For this purpose I use a novel description of the "Scri". This approach simplifies enormously description of gravitational energy and leads to a "Hamiltonian theory of radiation".

Vicent Pecastaing Université du Luxembourg

Conformal groups of closed Lorentzian manifolds and their dynamics

In the 1980's, Zimmer characterized the semi-simple part of the identity component of the isometry group of finite volume Lorentzian manifolds: it is either compact, or locally isomorphic to $\mathrm{SL}(2;\mathbb{R}) \times K$, where K is a compact semi-simple Lie group. A bit later, Gromov gave a very good description of closed Lorentzian manifolds on which a cover of $\mathrm{PSL}(2;\mathbb{R})$ acts by isometries, as well as its action. At the end of the 1990's, these works led Adams-Stuck and Zeghib to the classification of the full identity component of the isometry group of a compact Lorentzian manifold.

I will discuss the analogous problems for conformal transformations. One major change is that conformal maps *a priori* preserve no finite measure, and a richer family of examples appears. I will present recent results in this setting, mainly focused on conformal dynamics of semi-simple Lie groups, in the continuation of anterior investigations of Zimmer, Bader-Nevo and Francès-Zeghib. These results have consequences on the general form of the identity component of the conformal group of a compact Lorentzian manifold, and they partially confirm the Lorentzian version of a conjecture of Lichnerowicz.

Stefan Suhr Ruhr-Universität Bochum

Optimal transportation in Lorentzian geometry

Optimal transport has undergone a rapid development in recent decades with strong connections to Riemannian geometry, notably the introduction of weak notions of Ricci curvature in metric measure spaces. After introducing the optimal mass transportion problem in Lorentzian geometry I will discuss recent advances in the area.

Robert Wald University of Chicago

The Memory Effect in Higher Dimensional Spacetime

TBA

III. Contributed talks

Luis Alberto Aké Hau Universidad de Málaga

Splitting of globally hyperbolic spacetimes with timelike boundary

Globally hyperbolic spacetimes with timelike boundary, $(\overline{M}, g)$ where g is a Lorentzian metric and $\overline{M} = M \cup \partial M$, are the natural class of spacetimes containing naked singularities where boundary conditions can be posed; such conditions can be regarded as asymptotic, when the boundary is obtained by means of a conformal embedding. The properties of these spacetimes were introduced and studied in [1]. In here, we will study the properties of its causal ladder and we will focus in its top level, that is, *globally hyperbolic*. In particular, the well-known splitting of globally hyperbolic spacetimes without boundary is extended to a big class of causally continuous ones. This work is based on [2].

References

[1] D. Solís, *Global properties of asymptotically de Sitter and Anti de Sitter spacetimes*, Phd. Thesis, University of Miami, (2006).

[2] L. Aké, *Some global causal properties of certain classes of spacetimes*, Phd. Thesis, Universidad de Málaga, to be defended (2018).

Valter Borges Universidade de Brasilia

Ricci almost solitons on semi-Riemannian warped products

We characterize Ricci almost solitons on semi-Riemannian warped products, considering the potential function to depend on the fiber or not. We show that the fiber is necessarily an Einstein manifold. As a consequence of our characterization we prove that when the potential function depends on the fiber, if the gradient of the warping function does not act by translations then the base and the warped product are also Einstein manifolds. Moreover, we show the existence of conformal vector fields on the base, the fiber and on the warped product. Assuming completeness of the warped product we provide a classification of such manifolds. When the potential function depends on the fiber and the gradient of the warping functions is an improper vector field, we show that the base is a Brinkmann space and the fiber is Ricci flat. The proofs rely heavily on an important decomposition property of the potential function in terms of functions which depend on the basis or on the fiber. As an application of the characterization we also prove that the potential function of a complete Ricci soliton depends only on the base.

Joint wok with Keti Tenenblat

Ildefonso Castro-Infantes University of Granada

Curves in Lorentz-Minkowski plane with prescribed curvature

Motivated by a problem proposed by David A.Singer in 1999 and by the classical Euler elastic curves, we study in this talk spacelike and timelike curves in Lorentz-Minkowski plane $\mathbb{L}^2$. Our goal is to study the Singer's problem stated for the Lorentz-Minkowski plane; that is, to try to determine those curves $\gamma = (x, y)$ in $\mathbb{L}^2$ whose curvature κ depends on some given function $\kappa = \kappa(x, y)$. Concretely we study spacelike and timelike curves whose curvature is expressed in terms of the Lorentzian pseudodistance to fixed geodesics or to a fixed point. See [CCIs2] and [CCIs1].

Then, from a geometric-analytic point of view, we deal with the following problem: For a unit-speed parametrization of a regular curve $\gamma = (x, y)$ in $\mathbb{L}^2 := (\mathbb{R}^2, g = -dx^2 + dy^2)$

- We prescribe the curvature with the extrinsic conditions $\kappa = \kappa(\rho)$, where $\rho := \sqrt{|g(\gamma, \gamma)|} = \sqrt{|-x^2 + y^2|} \geq 0$.
- We prescribe the curvature with the extrinsic condition $\kappa = \kappa(v)$, where $v = y - x$.
- We prescribe its curvature with the extrinsic condition $\kappa = \kappa(y)$ or $\kappa = \kappa(x)$.

We get a complete description of the elastic curves in $\mathbb{L}^2$ and provide the Lorentzian versions of catenaries and grim-reaper curves. We also find out several new families of Lorentzian spiral. In addition, we provide uniqueness results for the generatrix curve of the Enneper's surface of second kind and for Lorentzian versions of some well known curves in the Euclidean setting, like the Bernoulli lemniscate, the cardioid, the sinusoidal spirals and some non-degenerate conics.

References

[CCIs1] I. Castro, I. Castro-Infantes and J. Castro-Infantes. *Curves in Lorentz-Minkowski plane: elasticae, catenaries and grim-reapers*. Preprint 2018.
[CCIs2] I. Castro, I. Castro-Infantes and J. Castro-Infantes. *On a problem of singer about curves in Lorentz-Minkowski plane.* Preprint 2018.

Ivan Costa e Silva Federal University of Santa Catarina (Brazil)

Hausdorff topology on the future causal boundary and a novel definition of null infinity

A natural Hausdorff topology τ_C is defined on the future causal completion $\hat{M}$ of a globally hyperbolic spacetime M with the following desirable features: the chronological future and pasts of points in $\hat{M}$ are open, future-directed sequences converge, M is a open dense set in $\hat{M}$ and the inclusion $i : M \to \hat{M}$ is open and continuous. In particular, the induced topology in M coincides with the manifold topology. The topology τ_C is shown to be strictly finer than the future chronological topology introduced in [1] and extensively discussed and champi- oned in

[2]. Inspired by recent work of O. Müller [3], this topology is compared to the future conformal boundary of M induced by a conformal extension $M \hookrightarrow \tilde{M}$ such that (i) $\tilde{M}$ is a globally hyperbolic spacetime, (ii) $M \subset I^{-}(K, \tilde{M})$ for some compact set $K \subset \tilde{M}$ and (iii) M is causally convex in $\tilde{M}$. In this context, it is shown that a homeomorphism exists which maps the conformal boundary onto the future causal boundary, so that both boundaries coincide and are homeomor- phic to Cauchy hypersurfaces in M. Finally, a new definition of *(future) null infinity* is defined which is independent of the existence of conformal boundaries, and some of its properties are described.

References

[1] J.L. Flores, *The Causal Boundary of spacetimes revisited*, Commun. Math. Phys. **276** (2007), 611-643.

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[3] O. Müller, *Which spacetimes admit conformal compactifications?*, to appear in Adv. Theor. Math. Physics (2017), arXiv:1409.8136.

Esma Demir Çetin Nevşehir Hacı Bektaş Veli University

Singularities of Developable Surfaces in Lorentz 3-Space

In this work we investigate singularities for three types of developable surfaces, introduced by Izumiya and Takeuchi, in Lorentz 3 space and give a local classification in terms of k-order frame [3,4]. Moreover we search the necessary conditions of being a geodesic for principal direction curves of the rectifying developable surface.

References

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[3] Izumiya, S., Takeuchi, N., *New special curves and developable surfaces*, Turk J. Math. **28** (2004), 153-163

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[5] Yaylı, Y., Saraçoğlu, S., *On developable ruled surfaces in Minkowski space*, Adv. Appl. Clifford Algebras **22** (2012), 499-510

Maciej Dunajski University of Cambridge

Solitons on the wormhole

TBA

José L. Flores University of Málaga

Ehlers-Kundt conjecture about Gravitational Waves and Dynamical Systems

Ehlers-Kundt conjecture is a physical assertion about the fundamental role of plane waves for the description of gravitational waves. Mathematically, it becomes equivalent to a problem on the Euclidean plane $\mathbb{R}^2$ with a very simple formulation in Classical Mechanics: given a non-necessarily autonomous potential $V(z,u)$, $(z,u) \in \mathbb{R}^2 \times \mathbb{R}$, harmonic in z (i.e. source-free), the trajectories of its associated dynamical system $\ddot{z}(s) = -\nabla_z V(z(s), s)$ are complete (they live eternally) if and only if $V(z,u)$ is a polynomial in z of degree at most 2 (so that V is a standard mathematical idealization of vacuum). Here, we show a proof of the conjecture in the significative case that V is bounded polynomially in z for finite values of $u \in \mathbb{R}$. The mathematical and physical implications of this *polynomial EK conjecture*, as well as the non-polynomial one, are discussed beyond their original scope.

References

[1] J.L. Flores, M. Sánchez, *Ehlers-Kundt conjecture about Gravitational Waves and Dynamical Systems*. Preprint 2017. Available at arXiv:1706.03855.

Melanie Graf University of Vienna

Rigidity of asymptotically $AdS_2 \times S^2$ spacetimes

The spacetime $AdS_2 \times S^2$ is well known to arise as the 'near horizon' geometry of the extremal Reissner-Nordstrom solution, and for that reason it has been studied in connection with the AdS/CFT correspondence. Following recent work with Greg Galloway ([1]) I will look at asymptotically $AdS_2 \times S^2$ spacetimes that obey the null energy condition (or a certain averaged version thereof). For such spacetimes one can recover some of the geometrical features of exact $AdS_2 \times S^2$, namely the existence of two transverse foliations by totally geodesic achronal null hypersurfaces, whose intersections are isometric to round 2-spheres. Even further rigidity can be established under the additional condition $\nabla Ric = 0$. It is well known that for Riemannian manifolds this is equivalent to the manifold being locally a product of Einstein manifolds and under the added assumption that the Ricci tensor is non-degenerate this remains true in the Lorentzian case. This allows one to show that any asymptotically $AdS_2 \times S^2$ spacetime satisfying the null energy condition and $\nabla Ric = 0$ is globally isometric to $AdS_2 \times S^2$.

References
[1] G. J. Galloway and M. Graf, *Rigidity of asymptotically $AdS_2 \times S^2$ spacetimes*, arXiv:1803.10529 (2018).

Stacey Harris University of St Louis

Towards an exploration of spacelike singularities in general spacetimes

It is possible to give a description of when a general stongly causal spacetime—with no assumed symmetries—has a geometry that results in a spacelike causal boundary. This geometric description is in terms of a foliation by timelike curves and of the behavior of the metric in the distribution perpendicular to the foliation. Being spacelike, this boundary comes imbued with a topology that all methodologies agree upon; this makes it ideal for exploring the topology of spacelike singularities. The conditions being given in geometric terms, we can explore how geometry in the spacetime may or may not detect geometric singularity in the boundary.

Jónatan Herrera Universidad de Córdoba

A generalized notion for black holes using the causal boundary

Classical definitions for black holes [2, 4] relies in having a *conformal boundary* defined. In fact, they consider the so-called *future null infinity* $\mathcal{J}^+$, a subset of the conformal boundary which plays the role of "escape area" for the black hole. That is, a point is not in the black hole if it can be connected with $\mathcal{J}^+$ by means of a causal curve. This approach presents however an important drawback: the conformal boundary is not available for all spacetime models. This is the case for instance, as pointed out by Marolf and Ross, for plane waves [3].

In order to circumvent this, we will present a new definition for black holes which makes use of the so called *causal boundary* [1]. This boundary is defined for any strongly causal spacetime, and so it will be suitable for models where no conformal boundary is available. We will show that most of the classical properties for black holes can be generalized for this new notion, including the fact that closed trapped surfaces are completely covered with black holes whenever the null convergence condition holds. Moreover, we will show that generalized plane waves have no black holes.

References
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[2] Stephen W. Hawking and George F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge Monographs on Mathematical Physics), Cambridge University Press, 1975.

[3] Donald Marolf and Simon F. Ross, *Plane waves: To infinity and beyond!*, Classical and Quantum Gravity **19** (2002), no. 24, 6289–6302.
[4] R.M. Wald, *General Relativity*, University of Chicago Press, 1984.

Mikołaj Korzyński Center for Theoretical Physics, Polish Academy of Sciences

Covariant and observer-independent approach to geometric optics in GR

I will present a covariant, observer-independent approach to geometric optics in GR. We consider observables like angular sizes, image distortions, trigonometric parallax, observed peculiar motions as well as the position and redshift drifts, registered by an observer for a given light source far away. The observables will be presented as functions of certain bi-local operators on the one hand and 4- velocities and 4-accelerations of the observer and source on the other. The bi-local operators are covarianty defined and depend only on the spacetime geometry along the line of sight. They can be expressed as functionals of the Riemann tensor. This way formalism clearly separates the dependence of the observables on the background geometry from the dependence on the motions of the observer and the source.

In the presentation I stress the geometric aspects of the derivation, including the properties of the null geodesic deviation equation. I will discuss the role of the parabolic subgroup of the Lorentz group and how it is related to the issue of observer invariance. The formalism presented in the talk is applicable to any spacetime geometry and has important applications in astrophysics. It can be used to extract information about the relativistic corrections to cosmological drift effects, observed peculiar motions and parallax. On a more theoretical side, it also allows to define observer-independent quantities describing the light propagation in any spacetime. The talk is based on developments of ideas from paper [1].

References

[1] Korzyński M and Kopinski J 2018 *Journal of Cosmology and Astroparticle Physics* **2018** 012 URL http://stacks.iop.org/1475-7516/2018/i=03/a=012

Martin Lesourd University of Oxford

Strenghtened light delay theorems

We strengthen or extend a number of existing theorems sharing the common feature of manipulating maximal non-spacelike geodesics (eg. lines, rays, etc.). Our aim is to highlight certain conceptual and mathematical points of interest by paying particular attention to connections with various other existing results, along with various parallels occurring with Riemannian geometry. The theorems include a splitting theorem of Galloway and Vega [Gall-Vega], the light delay theorems of Wald and Gao [Wald-Gao], and the cosmological singularity theorem of Galloway and Ling [Gall-Ling]. We do not make the a priori assumption global hyperbolicity but rather seek to derive it via connections occurring between the existence of maximal lines and various levels of the causal ladder. Our strengthening does not proceed as in the standard Hawking-Penrose singularity theorem, though indeed that methodology could be applied here. As in [Gall-Ling], we exploit certain recent advances in the understanding of the topology of 3-manifolds.

Jerzy Lewandowski University of Warsaw

Stationary isolated horizons of the Petrov type D

3-dimensional null surfaces that have local properties of Killing horizons are defined and called stationary to the second order isolated horizons (in short: stationary horizons). They are embedded in 4-dimensional spacetimes that satisfy the vacuum Einstein equations with arbitrary cosmological constant. Intrinsic geometry of stationary horizon consists of induced degenerate metric tensor and induced twist free connection. In the case of non-degenerate stationary horizon it determines its spacetime Weyl tensor. An assumption that the Weyl tensor is of the Petrov type D amounts to some complex equation on invariants of the intrinsic geometry. The equation is shown to be an integrability condition for the so called Near Horizon Geometry equation. The emergence of the Near Horizon Geometry in this context is equivalent to the hyper-suface orthogonality of the principal null direction of the Weyl tensor transversal to the horizon. In the case of bifurcated stationary horizon the type D equation implies the axial symmetry. In this way the axial symmetry is ensured without the rigidity theorem. We solve the type D equation in that case completely assuming that the bifurcation surface topologically is sphere. The family of solutions is 2-dimensional, and can be parametrized by the area and momentum. This is a local version of the no-hair theorem for type D bifurcated isolated horizons. For cross-sections of genus > 0 the Petrov type D equation is solved completely due to methods of the algebraic topology.

Verónica López Cánovas Universidad de Murcia

Codimension two spacelike submanifods through a null hypersurface of a pf-wave

A Brinkmann spacetime is a Lorentzian manifold $\overline{M}$ which admits a globally defined null vector field $K \in \mathfrak{X}(\overline{M})$ which is parallel. This family of spacetimes contains exact solutions to the Einsteins field equation that model radiation (electromagnetic or gravitational) moving at the speed of light. Lately, Brinkmann spacetimes have attracted a great deal of attention due to the experimental detec- tion of gravitational waves.

In this work we consider a relevant subfamily of these spacetimes, the plane- fronted waves (in sort, pf-waves). Concretely, we study codimension two spacelike submanifolds, obtaining some interesting results for the case when they are im- mersed in a pf-wave through a distinguished null hypersurface. As a new approach of these type of results, we also get some conditions which imply that a codimension two spacelike submanifold has to be immersed through such a null hypersurface.

This is part of a work in progress with Alfonso Romero (from Universidad de Granada) [1].

References

[1] V.L. Cánovas and A. Romero. *Mean curvature of codimension two spacelike submanifolds in certain Brinkmann spacetimes*. Preprint.

Tomasz Miller Warsaw University of Technology

Causal evolution of probability measures

Causal precedence relation J^+, defined as a binary relation on a given spacetime $\mathcal{M}$, can be quite naturally extended onto the space $\mathcal{P}(\mathcal{M})$ of Borel probability measures on $\mathcal{M}$. Using the tools of the optimal transport theory adapted to the Lorentzian setting, one can utilize thus obtained notion of 'causality between measures' to model a causal time-evolution of a spatially distributed physical entity in a globally hyperbolic spacetime. In my talk I will briefly present the abovementioned extension of J^+ onto $\mathcal{P}(\mathcal{M})$ and define what it means that a time-dependent probability measure $\mu_t \in \mathcal{P}(\mathcal{M})$ evolves causally. I will present how such an evolution can be understood as a 'probability measure on the space of worldlines'. I will also discuss some preliminary results concerning the relationship between the causal time-evolution of measures and the continuity equation.

References

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Lorenzo Nicolodi Università di Parma

Lorentz manifolds whose restricted conformal group has maximal dimension

We explicitly describe the restricted conformal group of the Einstein static universe of dimension $n + 1 \geq 3$ as a nontrivial central extension of the pseudo- orthogonal group of signature $(2, n + 1)$. This group has the maximal dimension permitted by a conformal Lorentz structure on a manifold of dimension $n + 1$. Two countably infinite series of conformally nonequivalent compact quotients of the Einstein static universe whose restricted conformal groups have maximal dimension are also considered. These Lorentz manifolds are referred to as the *integral compact forms*. We show that an oriented, time-oriented, conformal Lorentz manifold of dimension $n + 1 \geq 3$ has a restricted conformal group of maximal dimension if and only if it is conformally equivalent to either the Einstein static universe or to an integral compact form. This is a report on joint work with O. Eshkobilov and E. Musso (cf. preprint at `http://www2.unipr.it/~lnicolo9/LoMaRCGmaxD.pdf`).

Miguel Ortega University of Granada

Hopf Real Hypersurfaces in the Indefinite Complex Projective Space

This is joint work with Makoto Kimura (Ibaraki University, Japan) `https://arxiv.org/abs/1802.05556`. The study of real hypersurfaces in indefinite complex projective space seems to be initiated by A. Bejancu and K. L. Dugal in [2]. However, we had to wait for H. Anciaux and K. Pana- giotidou to obtain new results in [1] concerning non-lightlike hypersurfaces. In these papers, the authors link two classical branches of Differential Geometry, namely, real hypersurfaces in complex space forms and semi-Riemannian Geometry.

In this paper, we wish to further develop H. Anciaux and K. Panagiotidou's ideas, but we will just focus on the indefinite complex projective space $\mathbb{C}P^n_p$ of index $1 \leq p \leq n - 1$.

We reobtain the almost contact metric structure on a real hypersurface in $\mathbb{C}P^n_p$, namely (g, ϕ, ξ, η). We construct new families of real hypersurfaces, which we call *real hypersurfaces of type A_+, A_-, B_0, B_+, B_- and C*, because they are somehow similar to those in Takagi's and Montiel's lists ([5], [6]). All our examples are Hopf ($A\xi = \mu\xi$, A the shape operator). In particular, we positively answer the first open problem in [1]. We also exhibit an example of a lightlike Hopf real hypersurface.

J. Berndt in [3] and M. Kimura in [4] proved that a real hypersurface in a complex space form is Hopf and has constant principal curvatures if, and only if, it is one of the examples in Montiel's list and Takagi's list, respectively. Then, we hope that a similar result holds true in our setting.

Conjecture 1. *Let M be a non-degenerate real hypersurface in $\mathbb{C}P^n_p$ whose shape operator is diagonalisable .Then, M is Hopf and all its principal curvatures are constant if, and only, if, M is locally congruent to one of the examples A_+, A_-, B_0, B_+, B_-, or C.*

We prove that two real hypersurfaces with the same shape operator are linked by a holomor phic isometry of the ambient space. This was obtained by R. Takagi in [6] for real hypersurfaces in the Riemannian complex projective space. Next, we obtain the list of non-degenerate real hypersurfaces in indefinite complex projective space such that $AX = \lambda X + \rho\eta(X)\xi$, where λ and ρ are smooth functions. They were studied in complex space forms by S. Montiel, [5], and by R. Takagi, [7]. Finally, we solve the second open problem in [1], because we classify those non-degenerate real hypersurfaces such that $A\phi = \phi A$.

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José A. S. Pelegrín Universidad de Granada

Extending Calabi's duality

In 1970 Calabi discovered a nice correspondence between solutions of the minimal surface equation in the Euclidean space $\mathbb{R}^3$ and solutions of the maximal surface equation in the Lorentz-Minkowski spacetime $\mathbb{L}^3$ [1]. Later on, several authors have studied this duality be- tween minimal and maximal graphs using different approaches (see [2], for instance).

Concretely, in [3] we find a correspondence between solutions of the minimal surface equation in a Riemannian warped product with base a 2-dimensional Riemannian surface (B, g_B), fiber the real line and warping function $\frac{1}{\sqrt{\gamma}}$

$$\text{(R)} \qquad \operatorname{div}\left(\frac{Du}{\sqrt{\gamma + |Du|^2}}\right) = \frac{1}{2\gamma}\frac{g_B(D\gamma, Du)}{\sqrt{\gamma + |Du|^2}}$$

and solutions of the maximal surface equation in a standard static spacetime with the same base (B, g_B), fiber the real line endowed with the negative of its standard metric and warping function $\sqrt{\gamma}$, which is written as

$$\text{(L.1)} \qquad \operatorname{div}\left(\frac{D\omega}{\sqrt{\frac{1}{\gamma} - |D\omega|^2}}\right) = \frac{\gamma}{2}\frac{g_B\left(D\left(\frac{1}{\gamma}\right), D\omega\right)}{\sqrt{\frac{1}{\gamma} - |D\omega|^2}} \,,$$

$$\text{(L.2)} \qquad |D\omega|^2 \leq \frac{1}{\gamma} \,.$$

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Istvan Racz University of Warsaw

A new proof of the positive mass theorem

The positive mass theorem is proved for arbitrary spacelike slices, with non-negative scalar curvature and admitting quasi-convex foliations, in asymptotically flat Lorentzian spaces.

Çağla Ramis Nevşehir Hacı Bektaş Veli University

Transition Surfaces Foliated by Circles in Lorentzian Space

The transition surface is a kind of cyclic surface formed by a system of generating circles. The system of generating circles is consisted of a set of centers, radii and carries planes [1]. In Lorentzian three-space, three types of circles are defined depends on the planes. In this study, we introduce the transition surfaces foliated by Lorentzian circles in terms of the general radii functions with two parameters. We investigate the conditions for regularity of surfaces and give their structural properties obtained by fundamental forms and curvatures.

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Gabriel Ruiz Universidad Nacional Autonoma de Mexico (UNAM)

Timelike surfaces in Minkowski space with a canonical null direction

Given a constant vector field Z in Minkowski space, a timelike surface is said to have a canonical null direction with respect to Z if the projection of Z on the tangent space of the surface gives a lightlike vector field. For example in the three-dimensional Minkowski space: A surface has a canonical null direction if and only if it is minimal and flat. When the ambient has arbitrary dimension, if a surface has a canonical null direction and has parallel mean curvature vector then it is minimal. We give different ways for building these surfaces in the four-dimensional Minkowski space. On the other hand, we describe several properties in the non ruled general case in four-dimensional Minkowski space. We describe these surfaces locally.

Clemens Sämann Faculty of Mathematics, University of Vienna

Lorentzian length spaces

We introduce an analogue of the theory of length spaces into the set- ting of Lorentzian geometry and causality theory. The rôle of the metric is taken over by the time separation function, in terms of which all basic no- tions are formulated. In this way we recover many fundamental results in greater generality, while at the same time clarifying the minimal require- ments for and the interdependence of the basic building blocks of the theory. A main focus of this work is the introduction of synthetic curva- ture bounds, akin to the theory of Alexandrov and CAT(k)-spaces, based on triangle comparison. Applications include Lorentzian manifolds with metrics of low regularity, closed cone structures, and certain approaches to quantum gravity. If time permits, an application to the low regularity (in)-extendibility of spacetimes is given. This is joint work with Michael Kunzinger. Preprint: `https://arxiv.org/abs/1711.08990`

Yafet Sanchez Max Planck Institute Bonn

Green operator for low regularity spacetimes

In this paper we define and construct advanced and retarded Green operators for the wave operator on spacetimes with low regularity. In order to do so we require that the spacetime satisfies the condition of generalised hyperbolicity which is equivalent to well- posedness of the classical inhomogeneous problem with zero initial data where weak solutions are properly supported. Moreover, we provide an explicit formula for the kernel of the Green operators in terms of an arbitrary eigenbasis of H 1 and a suitable Green matrix that solves a system of second order ODEs. The talk is based on `https://arxiv.org/abs/1711.01842`.

Fábio Santos Universidade Federal de Campina Grande / Universidad de Murcia

Characterizations of Spacelike Submanifolds with Constant Scalar Curvature in the de Sitter Space

In this work, we deal with complete spacelike submanifolds M^n immersed in the de Sitter space $\mathbb{S}_p^{n+p}$ of index p with parallel normalized mean curvature vector and constant scalar curvature R. Imposing a suitable restriction on the values of R, we apply a maximum principle for the so called Cheng-Yau operator L, which enables us to show that either such a submanifold must be totally umbilical or it holds a sharp estimate for the norm of its total umbilicity tensor, with equality if and only the submanifold is isometric to a hyperbolic cylinder of the ambient space. In particular, when $n = 2$ this provides a nice characterization of the totally umbilical spacelike surfaces of $\mathbb{S}_p^{2+p}$ with codimension $p \geq 2$. Furthermore, we also study the case in which these spacelike submanifold are L-parabolic. This work was developed jointly with the researchers Luis J. Alías and Henrique F. de Lima.

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Didier Solis Universidad Autonoma de Yucatan

Classification of null hypersurfaces in Robertson Walker spacetimes

One of the most distinctive feature in Lorentzian geometry is the presence of null (degenerate) submanifolds. In this talk we address some recent developments in the classification of null hypersurfaces subject to geometric constraints immersed in Robertson-Walker spacetimes. In particular, we will focus on umbilical, isoparametric and Einstein null hypersurfaces. This is joint work with Oscar Palmas (UNAM) and Matias Navarro (UADY).

Roland Steinbauer Faculty of Mathematics, University of Vienna

The classical singularity theorems under optimal regularity conditions

The optimal regularity where the formulation of the classical singularity theorems of General Relativity make sense is $C^{1,1}$, i.e., the first derivatives of the metric being locally Lipschitz continuous. An extension of the theorems to this class is highly desirable also from a physical point of view: Via the field equations a $C^{1,1}$-metric corresponds to a finite jump in the matter variables thus constituting a viable model in GR, e.g. a matched spacetime.

While the Hawking and the Penrose singularity theorems have been extended to this regularity class [3, 4] by the use of regularisation techniques adapted to the causal structure [1], it remained unclear whether these techniques would also allow to prove the most general of the classical theorems. In fact the Hawking and Penrose theorem [2] relies on a more elaborate analysis of the occurrence of conjugate points.

We discuss the recent proof of this theorem for $C^{1,1}$-metrics [5]. The key ingredient turns out to be a refined analysis of the matrix Riccati equation for smooth metrics which leads to a focusing of causal geodesics even under mild violations of the energy conditions. We put special emphasis on these new estimates which are of independent interest, and we discuss further prospects opened by them.

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Arman Taghavi-Chabert American University of Beirut

Lorentzian geometry and CR structures

It is a remarkable fact of mathematical relativity that a shearfree foliation of null geodesics (SFNG) on a Lorentzian 4-manifold has an associated CR structure on its leaf space. Such foliations are of central importance in the study of solutions of Einstein's equations, and their existence restricts the Weyl conformal curvature as stated by the celebrated Goldberg- Sachs theorem. They also laid the foundations of Penrose' twistor theory. In the words of Nurowski and Trautman, such manifolds may be viewed as "Lorentzian analogues of Hermitian manifolds": they provide a playground for the interaction between Lorentzian geometry and complex geometry. In this talk I will examine how these notions generalise to dimensions other than four.

Juan A. Valiente Kroon Queen Mary University of London

Killing spinor data on non-expanding horizons and the uniqueness of vacuum stationary black holes

We make use of Racz's holograph construction to analyse the existence of Killing spinors in the domain of dependence of the horizons of distorted black holes. In particular, we provide conditions on the bifurcation sphere ensuring the existence of a Killing spinor. These conditions can be understood as restrictions on the curvature of the bifurcation sphere and ensure the existence of an axial Killing vector on the 2-surface. We obtain the most general 2-dimensional metric on the bifurcation sphere for which these curvature conditions are satisfied. In addition, we formulate further conditions on the bifurcation sphere ensuring that the Killing vector associated to the Killing spinor is Hermitian. Once the existence of a Hermitian Killing vector is guaranteed, one can use a characterisation of the Kerr spacetime due to Mars to identify the particular subfamily of 2-metrics giving rise to a member of the Kerr family in Racz's holograph construction. Our analysis sheds light on the role of asymptotic flatness and curvature conditions on the bifurcation sphere in the context of the problem of uniqueness of stationary black holes.

This is work in collaboration with I. Racz (Wigner Institute, Budapest) and Michael J. Cole (graduate student QMUL).

Yusuf Yaylı Ankara University

Mannheim Slant Helix in Lorentz-Minkowski Space

There are many studies about the associated curves such as Bertrand curve, Mannheim curve, etc. If there is a relationship between the curves α and β such that, principal normal lines of α coincide with the binormal lines of β, then this curves are called Mannheim pair. The well known characterization of a Mannheim curve by using the curvature and torsion functions is given by $\kappa = \lambda(\kappa^2 + \tau^2)$ where λ is a non-zero constant. Then a curve is called slant helix if its principal normal vector field makes a constant angle with a fixed direction. Then it is proved that a curve is a slant helix if and only if the function

$$\frac{\kappa^2}{(\kappa^2 + \tau^2)^{\frac{3}{2}}} \left(\frac{\tau}{\kappa}\right)'$$

is constant.

In this study, we give characterizations of the Mannheim curves in three-dimensional Lorentz-Minkowski space $\mathbb{L}^3$. The main goals of the talk are to introduce slant Mannheim curves which provide both slant helices and Mannheim curves properties and to investigate the characterizations of this kind of curves.

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Handan Yıldırım Istanbul University

About Legendrian non-flat dual surfaces of a spacelike curve in the lightcone

In this talk which is based on a joint work with Kentaro Saji, taking into account the Legendrian dualities in [2] which are extensions of the Legendrian dualities in [1], we first introduce new extended Legendrian dualities in Lorentz-Minkowski 4-space. Secondly, by means of these Legendrian dualities, we construct Legendrian dual surfaces in various radius pseudo-hyperspheres of a spacelike curve in the lightcone. Finally, we mention about the singularities of these surfaces in the cases that they are non-flat.

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IV. Poster session

Alma L. Albujer Brotons University of Córdoba

Uniqueness results for general static splittings

It seems natural to think that the existence of different splittings on a standard static model should be subject to strong geometric restrictions, since it should admit some special kind of symmetries. In fact, it is well-known the uniqueness of splitting in a spatially closed static spactime, as it was proved by Sánchez and Senovilla in [3] and by Aledo, Romero and Rubio in [2].

In this talk we present a new general approach for the study of spacetimes admitting a standard static splitting. This approach allows us to give an alternative proof in the spatially closed case. However, our technique also allows us to obtain new uniqueness results for standard static models with a complete (non necessarily compact) spacelike base under some mild hypothesis. Finally, we will also present a rigidity result for general static spacetimes admitting several standard static splittings, as well as a wide class of non-trivial examples of spacetimes admitting different standard static decompositions.

These results are part of a joint work with Jónatan Herrera and Rafael M. Rubio, and are contained in [1].

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Halyson Baltazar Universidade Federal do Piauí - UFPI

A new Calabi-Bernistein type result in spatially closed GRW spacetime

The aim of this poster is to explain about uniqueness of a complete spacelike hypersurface Σ^n with constant mean curvature H immersed in a spatially closed generalized Robertson-Walker (GRW) spacetime $\overline{M}^{n+1} = -I \times_f M^n$, whose fiber M^n has positive curvature (see [2, 3, 4, 5, 6] for more details about this subject). Supposing that the warping function f is such that $-\log f$ is convex along Σ^n and that $Hf' \leq 0$, we show that Σ^n must be isometric to a totally geodesic slice of $\overline{M}^{n+1}$. When $\overline{M}^{n+1}$ is a Lorentzian product space (i.e., a GRW spacetime with constant warping function), we obtain a new Calabi-Bernstein type result concerning the maximal spacelike hypersurface equation. This result is contained in [1].

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Magdalena Caballero University of Córdoba

On Galilean and relativistic spacetimes with certain symmetries

In this poster we revisit the concepts of conformal and spatially conformal Leibnizian vector fields. We study irrotational and spatially conformal Leibnizian fields of observers in symmetric Galilean spacetimes, as well as irrotational and spatially conformal Killing fields of observers in relativistic spacetimes. We find conditions under which one of those fields is conformal Leibnizian or conformal Killing, respectively. And finally we get characterizations of GGRW spacetimes and GRW spacetimes.

This is a joint work with Daniel de la Fuente and Rafael M. Rubio.

Antonio Wilson Cunha Universidade Federal do Piauí

On the first stability eigenvalue of closed submanifolds in the Euclidean and hyperbolic space

In this poster, we would like to present upper bounds for the first eigenvalue of the strong stability operator of a closed submanifold M^n, $n \geq 4$, immersed with parallel mean curvature vector field either in the Euclidean space $\mathbb{R}^{n+p}$ or in the hyperbolic space $\mathbb{H}^{n+p}$, in terms of the mean curvature and the length of the total umbilicity operator Φ of M^n. In particular, under a suitable constraint on $|\Phi|$, we guarantee that such a submanifold must be strongly unstable.

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Chang-Wan Kim Division of Liberal Arts and Sciences, Mokpo National Maritime University

Finsler manifolds without conjugate points

In this poster we present that the integral of the Ricci curvature on the unit tangent bundle SM of a complete Finsler manifold M without conjugate points is nonpositive and vanished only if M is flat, provided that the Ricci curvature on SM has an integrable positive or negative part. In particular, any Finsler torus without focal points is flat. Our method is based on comparison techniques on an associated Riccati equation and Birkhoff's Ergodic Theorem applied to the geodesic flow on the unit tangent bundle SM.

Martin Lesourd University of Oxford

A new collapse singularity theorem permitting chronology violation

A new theorem describing singularities and chronology violation in the context of black holes is formulated. The premise of the theorem involves various semi-global geometric conditions that capture features usually associated with black holes: a region containing future trapped surfaces that is bounded by a future achronal boundary generated by future complete null geodesics. After describing the specific conditions and the sense in which they may be natural, it is shown that a region corresponding to the black hole interior contains future null incomplete geodesics when the causal structure of the interior permits certain kinds of chronology violation. The precise causality condition is phrased in terms of Minguzzi's recent definition of the boundary of a chronology violating class. Certain new features of the theorem include the ability to bypass the generic (timelike or null) conditions and the ability to locate future incomplete null geodesics within a region corresponding to the black hole interior.

Josef Šilhan Masaryk University, Brno

Conformal theory of curves with tractors

We present the general theory of curves in conformal geometry using tractor calculus. This primarily involves a tractorial determination of distinguished parametrizations and relative and absolute conformal invariants of generic curves. The absolute conformal invariants are defined via a tractor analogue of the classical Frenet frame construction and then expressed in terms of relative ones. This approach applies likewise to conformal structures of any signature; in the case of indefinite signature we focus especially on the null curves. It also provides a conceptual tool for handling distinguished families of curves (conformal circles and conformal null helices) and conserved quantities along them.

This is a joint work with V. Zadnik.

Lenka Zalabova Masaryk University and University of South Bohemia

Notes on symmetric conformal geometries

We summarize the results on symmetric conformal geometries. We review the results following from the general theory of symmetric parabolic geometries and prove several new results for symmetric conformal geometries. We show that each symmetric conformal geometry is either locally flat or covered by a pseudo-Riemannian symmetric space, where the covering is a conformal map. We construct examples of locally flat symmetric conformal geometries that are not pseudo-Riemannian symmetric spaces.

The talk is based on `https://arxiv.org/abs/1503.02505`.
