UCLA College Physical Sciences

Department of Mathematics
520 Portola Plaza
Box 951555
Los Angeles, CA 90095-1555

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dr. hab. Piotr Nowak, Scientific Director
Institute of Mathematics
Polish Academy of Sciences

Dear Dr. Nowak,

I am writing on your request for an evaluation of the habilitation thesis of Dr. **Grigor Sargsyan**.

Let me start by saying that Dr. Sargsyan is easily the foremost expert, worldwide, on descriptive inner model theory. His reputation as a leading researcher was established early on, through his thesis, that expanded the the core model induction method to levels that had previously been completely beyond reach. His thesis was a paradigm shift in the area, and earned him several awards including the Sacks Prize for the best dissertation in mathematical logic.

His work after the Ph.D. maintained the same exceptionally high standards of achievements and ingenuity, and he has continued to be a leader in the field, constantly pushing the core model induction, and descriptive inner model theory, to new heights.

In particular, the specific papers that form his habilitation thesis are groundbreaking on several fronts.

The papers (1) and (4), covering with universally Baire operators, and covering with Chang models over derived models, introduce a new abstract ways to view the question of covering, which is one of the key goals of the constructions of inner models, and an essential step in applications to consistency results. In (1), Sargsyan phrases a conjecture that incorporates several covering options for hybrid mice. Each of the options would typically suffice for applications to consistency strengths. He then conjectures that either one of these options holds, or else there is an inner model with superstrong cardinals, a very substantial large cardinal axiom, and the ultimate (but still far away) goal of many key questions in descriptive inner models theory. In (4), something related is done but now the covering model is a model of determinacy, obtained as a symmetric extension with the novel addition of all countable subsets of a fixed ordinal. In both cases he shows that the covering conjectures would have substantial applications to the central consistency strength goals of inner models theory. While the conjectures themselves remain open, he proves them in some settings: he shows that the conjecture from (1) holds up to a very high level of the determinacy hierarchy ($\mathsf{AD}_{\mathbb{R}} + \Theta$ regular), and he shows that the conjecture from (2) holds in HOD mice.

The paper (2), Translation procedures in descriptive inner model theory, develops a general translation mechanism between the standard inner models for large cardinals (called *mice*), and the more sophisticated ones that incorporate iteration strategies and capture the HOD inner models of universes satisfying determinacy (called HOD-*mice*). This allows him to translate strength from determinacy to standard large cardinal axioms. As an application he answers a question of Wilson on the consistency strength of the existence of stationarily many λ for which the derived model reaches at least the second step in the Solovay hierarchy. One direction of the consistency result was known, due to Woodin. Sargsyan's work provides the other direction (the one involving descriptive inner models theory, which has been open for a longer period).

The papers (3) and (6), Tame failures of the unique branches hypothesis and models of $\mathsf{AD}_{\mathbb{R}} + \Theta$ is regular, and Non-tame mice from tame failures of the Unique Branches Hypothesis, address the *Unique Branches Hypothesis* (UBH), a central statement in inner models theory, about the existence of cofinal branches through iteration trees, leading to well-founded direct limits. For the purpose of applications in inner model theory, there is no harm in imposing various niceness restrictions on the iteration trees. Tameness in this context is one such restriction. The hypothesis still fails for tame trees, by work of Woodin. But the work in these papers shows that such a failure requires large cardinals. Paper (6), joint with Nam Trang, shows that a failure of UBH for tame trees requires at least a model of determinacy reaching the second step in the Solovay hierarchy. The paper (3) (by Sargsyan as single author) improves this and shows that for such a failure one needs to go essentially as far as the current methods of inner models theory can reach.

The paper (5), Nontame mouse from a failure of square at a singular strong limit, address one of the key questions in inner models theory, the consistency strength of a failure of the combinatorial principle square. This question has a very long history, and is especially important, because square plays a role in many consistency results, including for example the results on the proper forcing axiom. Sargsyan shows that a failure of square requires reaching at least the second step in the Solovay hierarhcy. This is the best result on this question known to date.

Overall, the papers provide a substantial body of contributions to mathematics, at the forefront of research in descriptive inner models theory. They are original, detailed, deep, and complex. In many cases they obtain the best results currently known, further solidifying Sargsyan reputation as the leading researcher in his field. I think they combine to make for a habilitation of the highest caliber.

Sincerely,

I. Neeman

Prof. Itay Neeman
UCLA Mathematics
ineeman@math.ucla.edu