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**Report on the habilitation application of Dr. Grigor Sargsya**

Sargsyan work is at the juncture of the two most important research programs of Set Theory in the last two or three decades. The first program is the inner models theory. Facing the challenge of independence in Set Theory and the multitude of different possible universes of sets, the attention was drawn to models which has some canonical structure and which allow detailed analysis of their properties. The first such model is the constructible universe $L$, introduced by Gödel in 1938. $L$ allows a very detailed analysis of its structure and essentially every independent problem is settled by assuming that we live in $L$. The main drawback of adapting the assumption that $L$ is our universe of sets is that it is incompatible with axioms of strong infinity ("Large cardinals ") considered to be desirable. The challenge of the theory became getting canonical inner models , with $L$ like structure , which satisfy large cardinals hypothesis as strong as possible. This program has its successes ("models up to Woodin limit of Woodin cardinals") but still faces major challenges.

The other research program is the study of regularity properties of definable sets of reals . It turned out that the ultimate regularity property is always connected with the determinacy of infinite games. Determinacy means that one of the players in the given infinite game has a winning strategy. The Axiom of Determinacy -$AD$ introduced by Mycielski and Steinhaus, claims that every game where the moves are natural numbers is determined. It is incompatible wiht the axiom of choice , but it may hold for some definable inner model. $AD_R$ is the much stronger axiom claiming that all infinite games in which the games are reals are determined.

One of the spectacular achievements in Set Theory was the connection between large cardinals assumptions and determinacy. Large cardinals imply that natural inner models satisfy determinacy and models of determinacy involve canonical inner models for large cardinals. The fusion of theory of canonical inner models for large cardinals with the Descriptive Set Theory of reals became known as "Descriptive Inner Models Theory" .Grigor Sargsyan is one of the central figures of this emerging theory. It will be difficult to summarize in this letter all the major contributions of Sargsyan to the Descriptive Inner Models theory . Also I assume that you expect me to concentrate on the five papers

that are listed in the habilitation application, but I can not resist mentioning one major breakthrough of Sargsyan, which is not listed in the application.

A central conjecture of the theory is known as the Mouse Set Conjecture, which roughly claims that for any two reals $x$ and $y$, if $x$ is ordinal definable from $y$ then there is a very canonical way of constructing $x$ from $y$. ("$x$ belongs to some mouse over $y$".).In a tour de forcer work, published as a monograph in the Memoires of the American Mathematical Society., Sargsyan showed that the failure of the Mouse Set Conjecture will imply an inner model with rather strong theory ("$AD_R+\Theta$-regular.) Hence the conjecture holds in an interesting class of inner models.

Since the publication of this seminal monograph, Sargsyan produced a very impressive corpus of results. Let me concentrate on the papers listed in the application . The hierarchy of large cardinals is used to classify the consistency strength of different set theoretic statements. A related measure is the strength of determinacy model that one can get from a given combinatorial statement. A holy grail of this class of problems is the problem of determining the consistency strength of the failure of a combinatorial principle known as "Square" at a singular strong limit cardinal $\kappa$. ($\Box_\kappa$ at a cardinal $\kappa$.). A result of Steel is that the failure of $\Box_\kappa$ at a strong limit singular $\kappa$ implies that $AD$ holds in $L(R)$. The expectation is that the strength of it is much higher. The first step in showing it was done by Sargsyan. In the paper "Non tame mouse.." he proves that the failure of square at successor of singular strong limit singular is substantially stronger than $L(R)$ satisfying $AD$.

Similar significant progress on determining consistency strength of important statements is done in the two listed papers, which are joint papers with N. Trang. In these two papers they show that the failure of a principle which is important for developing the inner models theory, implies the consistency of $AD_R+\Theta$- regular. This is a major progress on what was known before. The other listed papers introduce two important conjectures : "Covering with Chang models" and "Covering with universally Baire operators". For both conjectures Sargsyan proves some very deep and significant consequences and shows that they hold in an interesting class of inner models. These two papers are the first steps to a very fruitful research program.

There is a common theme that goes through these five papers: it is the development of the Core Model Induction technique past $L(R)$. It is a technique that became very important tool for showing that a particular combinatorial statement , which superficially has nothing to do with large cardiaes or determinacy models, implies the existence of inner models with strong versions of the Determinacy. Sargsyan is one of the central developers and practitioners of this technique , and the present papers are a major contribution to this important tool.

In summary Grigor Sargsyan is a first rate researcher in Set Theory. His result are very deep and will no doubt will have an extensive impact on the field. The

collection of papers listed in habilitation application is a very impressive corpus of results which definitely qualifies for the degree of habilitated doctor. Also Sargsyan is very active in the scholarly community, organises conferences and seminars and he is an excellent tutor for graduate students. I attended several series of talks by him and were first rate: motivated clear and organized. I recommend in the strongest possible terms that scientific council will award Dr. Grigor Sargsyan the habilitation with all the rights that this degree implies.

Yours

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