

Abstract

The thesis is divided into three parts. We begin by introducing the reader to the topic of symmetric functions, and later move on to study the main object of focus in the thesis: *cumulants*. We conclude by proving a special case of an open conjecture concerning LLT cumulants.

In the first part, which consists of two chapters, we define the space of symmetric functions, as well as introduce classical notions and results in the theory. We highlight the connections to other branches of mathematics, such as representation theory or algebraic geometry to give motivation to studying symmetric functions. Most importantly, we mention the conjecture of Dołęga on Schur positivity of Macdonald cumulants, which was the starting point behind the results presented in the thesis.

The second part introduces a new class of symmetric functions: *LLT cumulants*, which aim to serve as a means to prove the conjecture of Dołęga. We define different normalizations of the cumulants and prove that Macdonald cumulants have a positive expansion in terms of LLT cumulants of ribbon shapes. Also, we interpret LLT cumulants as a weighted generating function of certain graph colorings, which proves useful in the study of the cumulants and allows for proving a number of positivity results, generalizing several recent advances in the field.

The last part gives a proof of a combinatorial formula for a positive LLT expansion of the LLT cumulant in the case when the sequence of shapes corresponds to a class of graphs called melting lollipops. In order to achieve that, we define Schröder paths and Schröder path relations which help understand the structure of unicellular LLT cumulants and, as a consequence, give an algorithmic way of decomposing LLT cumulants of melting lollipops.