Janusz Wyrwał
Silesian University of Technology
Institute of Automatic Control
Measurement and Control Systems Group

Controllability measures and their relations to some aspects of control theory

It happens very often in practice that the system is required to be controlled such that at some terminal time a prescribed final state, which is the target, is to be achieved. To answer the question if any admissible control exists that permits the desired terminal state to be achieved the analysis of controllability of dynamical system has to be performed.

The concept of controllability was introduced by Kalman as a tool for analysing behaviour of linear control systems. In general, the dynamical control system is said to be controllable on a time interval $[t_0, t_1]$ if, for given t_0 and t_1, each initial state can be transferred to any terminal state using admissible controls over the interval $[t_0, t_1]$. It should be emphasized that controllability means the ability to achieve a target state, but it does not imply that the reached state will be maintained. The question of keeping system in an achieved state is a separate problem.

Controllability analysis plays an important role in integrated approaches to design and control dynamical systems. It is strongly related to the fact that uncontrollable states can be described in terms of uncontrollable eigenmodes that can be excited by the initial state or by disturbances but cannot be affected through control. When system is not controllable, it is observed that only states contained in particular subspace of the state space can be controlled.

In the references relative to the different fields of control theory there exist a lot of publications devoted to controllability. Most of them analyse the problem of controllability of different classes of dynamical systems formulating qualitative controllability conditions. However, in practice it is important to have the ability of evaluating controllability quality of dynamical systems in a quantitative manner since it determines whether the system can be controlled effectively by a feedback control strategy. Therefore, on the viewpoint of engineering, it would be important to assess a controllability quality which explicitly indicate how easily to assign the poles of the system and achieve the target state with minimum expenditure of energy. These consideration leads to the notion of controllability measure since on the viewpoint of engineering, it seems to be too rough to classify all dynamical systems into only two disjoint categories: controllable and uncontrollable.

The main contribution of the present paper is to extend the understanding of relationships between quantitative controllability measures and main issues of mathematical control theory. In this paper, different quantitative criteria of controllability quality are analysed and compared. The relationships between pole assignment, minimum energy control and controllability measure are demonstrated. It is shown that controllability measure strongly influence magnitude of feedback gain required to assign poles and minimum energy expenditure of control that drives system to a desired target state to be achieved. Considerations are restricted entirely to linear systems although some of presented results may be generalized for broader class of dynamical systems.