Wawrzyniec Szatanik, M.Sc.
Gdańsk University of Technology
Department of Differential Equations

Solutions for generalized second order differential equations with deviating arguments

I wish to introduce my results in finding minimal and maximal solutions of generalized second order differential equations with deviating arguments.

Generalized second order differential equations with deviating arguments is defined by

$$\begin{cases} x''(t) = f\big(t, x(t), x(\alpha(t))\big), \quad t \in J = [0,T],\ T < \infty, \\ g_1\big(x(0), x(\delta_1), x(\delta_2), \dots, x(\delta_r)\big) = 0,\ \text{where } \delta_i \in (0,T), \\ g_2\big(x(T), x(\gamma_1), x(\gamma_2), \dots, x(\gamma_p)\big) = 0,\ \text{where } \gamma_i \in (0,T) \end{cases} \tag{1}$$

where $f \in C(J \times \mathbb{R} \times \mathbb{R}, \mathbb{R})$, $g_1 \in C(\mathbb{R}^{r+1}, \mathbb{R})$, $g_2 \in C(\mathbb{R}^{p+1}, \mathbb{R})$, p, $r \in \mathbb{N}$,

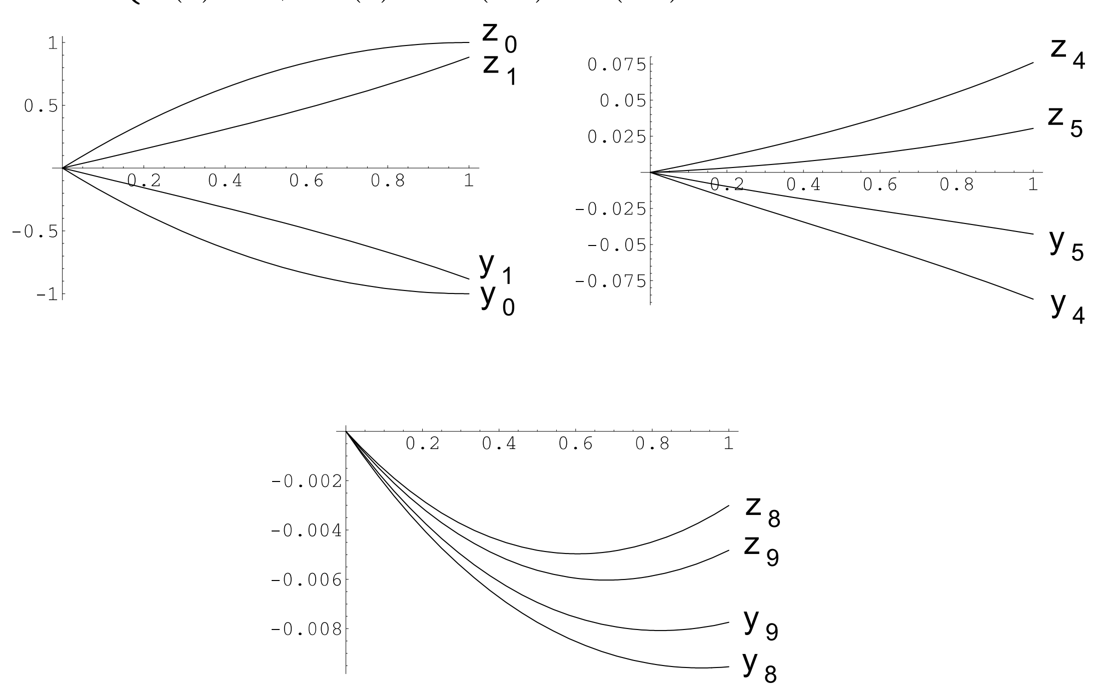
$$0 < \delta_1 < \delta_2 < \ldots < \delta_r < T, \quad 0 < \gamma_1 < \gamma_2 < \ldots < \gamma_p < T$$

and $\alpha \in C(J, J)$.

Solutions are found by monotone sequences of lower and upper solutions of (1).

Below we present the result of some numerical iteration of algorithm with is used in main theorem. This is the result for the problem

$$\begin{cases} x''(t) = \sin(t)x(t) + \cos(t)x(0.5t) + \frac{1}{32}, \quad t \in J = [0,1] \\ x(0) = 0, \quad x(1) = x^2(0.3) + x(0.4). \end{cases} \tag{2}$$



Chosen pairs of lower and upper solutions of (2)