dr hab. Ewa Schmeidel
dr inż. Zenon Zbąszyniak
Politechnika Poznańska
Instytut Matematyki

An application of Darbo's fixed point theorem in investigation of periodicity of solutions of difference equations

Using the technique of Darbo's fixed point theorem we investigate a nonlinear second order difference equation of the form

$$\Delta(r_n \Delta x_n) = a_n f(x_{n+1}) \tag{1}$$

where $n \in \mathbb{N}_0 := \{0, 1, 2, \ldots\}$, $x : \mathbb{N}_0 \to \mathbb{R}$, $a, r : \mathbb{N}_0 \to \mathbb{R}$, and $f : \mathbb{R} \to \mathbb{R}$ is continuous function.

Putting $f(x) = x$ in equation (1) we get Sturm-Liouville difference equation of the form

$$\Delta(r_n \Delta x_n) = a_n x_{n+1}. \tag{2}$$

The notion for an asymptotically ω-periodic function is given by the following definition.

Definition. Let ω be a positive integer. The sequence $y : \mathbb{N}_0 \to \mathbb{R}$ is called *ω-periodic* if $y(n+\omega) = y(n)$ for all $n \in \mathbb{N}_0$. The sequence y is called *asymptotically ω-periodic* if there exist two sequences $u, v : \mathbb{N}_0 \to \mathbb{R}$ such that u is ω-periodic, $\lim\limits_{n\to\infty} v(n) = 0$ and

$$y(n) = u(n) + v(n) \quad \text{for all } n \in \mathbb{N}_0.(3)$$

Sufficient conditions for the existence of an asymptotically periodic solution of this equation are obtained. An example illustrate the result. Next, the conditions under which this equation has no asymptotically periodic solutions are presented.