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On retrial queues controlled by the hysteresis strategy

The main characteristic of a retrial queues is that a primary customer who finds busy the service facility upon arrival immediately leaves the servers area, but some time later he repeats his demand (see, for example, [1]). Between retrials he is said to be in 'orbit'. Such queueing systems with repeated attempts have wide practical use in designing local area networks, communication network.

In this work we consider a retrial queue of type $M|M|1$ in which the number of retrials of each customer is limited by one attempt and which a rate of primary call flow depends on the number of customers in the orbit. A rate of the input flow are controlled by hysteresis strategy. Such strategy is built by means of the two thresholds H_1 and H_2, $H_1 \leq H_2$. If in some moment in time the number of customers in the orbit in the systems do not exceed H_1 then the system works in the first mode with rate of input flow h_1. If time the number of customers in the orbit exceed H_2 then the system works in the second mode with rate of input flow h_2. If the number of repeated calls is in the interval $(H_1, H_2]$ than the queue follows the mode in which it was at previous moment of time.

The system state at time t can be described by means of a trivariate process $Q(H_1,H_2,t) = \{(Q_1(H_1,H_2,t); Q_2(H_1,H_2,t); R(H_1,H_2,t)); t \geq 0\}$, where $Q_1(H_1,H_2,t)$ is the number of busy servers, $Q_2(H_1,H_2,t)$ is the number sources for one repeated attempt at time t and $R(H_1,H_2,t)$ is the system operating mode. The process $Q(H_1,H_2,t)$ is a continuous time Markov chain with the lattice semi-strip $S(Q(H_1,H_2,t)) = S^1(Q(H_1,H_2,t)) \bigcup S^2(Q(H_1,H_2,t))$ as the state space, where $S^1(Q(H_1,H_2,t)) = \{i = (i_1,i_2,1) : i_1 = 0,1; i_2 = \{0,\ldots,H_2\}\}$, $S^2(Q(H_1,H_2,t)) = \{i = (i_1,i_2,2) : i_1 = 0,1; i_2 = \{H_1+1,\ldots\}\}$.

In this work we consider optimization problem consisting in finding such thresholds H_1 and H_2, which maximize average income of system: $C_1S_1(H_1,H_2) - C_2S_2(H_1,H_2) - C_3S_3(H_1,H_2) \rightarrow \max$, where $S_1(H_1,H_2)$ – the number of calls that were served; $S_2(H_1,H_2)$ – the number of calls that become repeated calls; $S_3(H_1,H_2)$ – the number of switching of the input flow; C_1 – profit associated with service of a call; C_3 – penalty for refusal to serve; C_4 – penalty for switching rate of the input flow. The similar optimization problems for retrial queues have been considered in [2].

References

[1] G. I. Falin, J. G. C. Templeton, *Retrial Queues*, Chapman and Hall, London 1997.
[2] A. N. Dudin, V. I. Klimenok, *Optimization of dynamic control of input load in node of informational-computing network*, Automation and Computing Technology 1991, no. 2, 25–31.