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Analysis of biochemical reactions models with delays

Recently, there appears an increasing number of papers devoted to the description of gene regulatory processes, both from the experimental and theoretical point of view. During some gene regulatory processes, like transcriptional regulation there can appear oscillations which is supported by recent experiments. It is supposed that such oscillations can be delayed-induced. Time delays can play a significant role especially in circadian rhythms (e.g. in *Neurospora*, *Drosophila* and others) because they are long comparing to other time scales of the system. Thus, it would make sense to suspect that time delays being of the order of other time scales can also play an important role in the system dynamics. However, it is much more difficult to catch this type of behaviour experimentally.

We study three biochemical reaction channels starting from simple delayed protein degradation, through negative feedback with delayed production, till negative feedback with dimerisation that was considered by Bratsun *et al.* [3]. For the simple delayed degradation model, we have found that the form of both stochastic and deterministic descriptions used in [3] suggest that proteins which started the delayed degradation can still be involved in both type of degradation. This can eventually lead to the negative values of the solution (see [2]). We proposed an alternative description based on the mass action law and calculated explicit solution to this model.

For the model of reaction with negative feedback and delayed production, global stability of a unique positive steady state is proved. According to the models of these two reaction channels considered in the present paper there cannot appear delayed induced oscillations.

For the model of reaction with negative feedback, dimerisation and delayed protein production, local stability for a unique positive steady state is shown for some range of parameters. It is also proved that for some range of parameters the destabilisation due to the increasing delay can occur and delayed induced oscillations may appear.

References
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