Causal sets for geometrical Gandy machines and Gandy machines over multisets

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A new approach to the computational complexity beyond the known complexity measures of the consumed time and space of computation is proposed. The approach focuses on the chaotic behaviour and randomness aspects of computational processes and bases on a representation of these processes by causal sets. The abstract systems allowing some synchronized parallelism of computation are investigated within the approach, where the computational processes realized in a discrete time are represented by causal sets like in [2].

The representation of computational processes by causal sets is aimed to provide an abstraction from those features of computational processes which have not a spatial nature such that the abstraction could make visible some new aspects of the processes like an aspect of chaotic behaviour or a fractal shape.

The aspects of a chaotic behaviour and a fractal shape inspired by the research area of dynamics of nonlinear systems [11] regarding an unpredictability of the behaviour of these systems[1] could suggest an answer to the following question formulated in [13]: *Is the concept of randomness, founded in the concept of absence of computable regularities, the only adequate and consistent one? In which direction, if any, should one look for alternatives?*

The answers may have an impact on designing pseudorandom number generators applied in statistics, cryptography, and Monte Carlo Method.

Thus the proposed approach comprises measuring of complexity of computational processes by a use of graph dimensions [6] and network fractal dimensions [10] in parallel to measuring complexity of random strings in [4] by Hausdorff dimension.

The proposed approach concerns investigations of abstract computing devices of two types:

- geometrical Gandy machines,
- Gandy machines over multisets, where these machines are counterparts of some uniform families of P systems (the underlying systems of membrane computing [9]) like in [5].

The geometrical Gandy machines are some modifications of the known Gandy's mechanisms [3] by assuming that the sets of machine instantaneous descriptions[2] are skeletal sets, similarly like in [7], with respect to the permutations of urelements, where urelements are n-tuples of rational numbers. Hence the adjective "geometrical" is used.

It has been pointed out in [12] that Gandy mechanisms "conform to basic principles of physics", see also [1].

The geometrical Gandy machines are aimed to respect a claim (contained in the open problem in [12]) for Gandy mechanisms to be "more consistent with local causation in physics". In other words, to be

[1] unpredictability due to sensitive dependence on initial conditions—an important feature of deterministic transient chaos [11] often having fractal shape.

[2] a machine instantaneous description is here a hereditarily finite set which describes potential 'wireless' intercommunication between urelements appearing in this set.

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more realistic than (possible) imaginary constructs within the theory of hereditarily finite sets and hence less “technically somewhat complicated”.

The Gandy machines over multisets are geometrical Gandy machines, where the urelements appearing in the machine instantaneous descriptions are in addition labelled by multiset and the machine rules of local causation respect processing of multisets in the manner of P system evolution rules. These machines are aimed to present some uniform families of P system [5] in terms of a one machine with finite number of schemes of evolution rules.

The assumption that the sets of machine instantaneous descriptions are skeletal together with the features of machine local causation rules provide a natural construction of causal sets representing computational processes. The causal sets representing computational processes are here subsets of space-time, i.e. $(n+1)$-tuples of rational numbers if urelements are n-tuples (forming phase space), and the causality relations are determined by the applications of machine local causation rules, respectively.

The examples of Gandy–Păun–Rozenberg machines, including generalized machines, in [7] and [8] give rise to the examples of geometrical Gandy machines after some slight modifications.

Some models of computation in [2] may give rise to examples of geometrical Gandy machines whose computational processes are represented by causal sets of fractal shape.

References

[1] P. Arrighi & G. Dowek (2011): *The physical Church–Turing thesis and principles of quantum theory*. arXiv: 1102.1612.

[2] T. Bolognesi (2010): *Causal sets from simple models of computation*. Int. Journal of Unconventional Computing 6, pp. 489–524.

[3] R. Gandy (1980): *Church's thesis and principles for mechanisms*. In: The Kleene Symposium, North-Holland, Amsterdam, pp. 123–148.

[4] J. H. Lutz (2003): *The dimensions of individual strings and sequences*. Information and Computation 187, pp. 49–79.

[5] N. Murphy & D. Woods (2011): *The computational power of membrane systems under tight uniformity condition*. Nat. Computing 10, pp. 613–632.

[6] T. Nowotny & M. Requardt (1998): *Dimension theory of graphs and networks*. J. Phys. A Math. Gen. 31, pp. 2447–2463.

[7] A. Obtułowicz (2011): *Randomized Gandy–Păun–Rozenberg machines*. In: Membrane Computing, Lecture Notes in Computer Science 6501, Springer, Berlin, pp. 305–324.

[8] A. Obtułowicz (2012): *Generalized Gandy–Păun–Rozenberg machines for tile systems and cellular automata*. In: Conference on Membrane Computing 2011, Lecture Notes in Computer Science 7184, Springer, Berlin, pp. 314–322.

[9] G. Păun, G. Rozenberg & A. Salomaa (2009): *The Oxford Handbook of Membrane Computing*. Oxford Univ. Press, Oxford.

[10] H. D. Rozenfeld, L. K. Gallos, Ch. Song & H. A. Makse (2009): *Fractal and transfractal scale-free networks*. In: Encyclopedia of Complexity and System Science, Springer, Berlin, pp. 3924–3943.

[11] S. H. Strogatz (1994):, *Nonlinear Dynamics and Chaos*. Perseus Books Publ., LLC.

[12] K. Sutner (1998): *Computation theory of cellular automata*. In: MFCS'98 Satellite Workshop on Cellular Automata, Brno, Czech Republic, August 25–27, 1998.

[13] S. B. Volchan (2002): *What is a random sequence*. Amer. Math. Monthly 109, pp. 46–63.