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On the Hausdorff dimension of the attractor of a two-dimensional skew product with a tent map in the base

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Abstract. Let $\alpha \in \left[\sqrt{2}, 2\right]$, and define the tent map $T : [0,1] \to [0,1]$ by $Tx := \frac{\alpha}{2} - \alpha \left|x - \frac{1}{2}\right|$. Assume that $g(x,y) := \varphi(x) + \lambda \left(y - \frac{1}{2}\right)$ for some $\lambda \in \left(0, \frac{1}{\alpha^2}\right)$ and for a linear map $\varphi : [0,1] \to \left[\frac{\lambda}{2}, 1 - \frac{\lambda}{2}\right]$. Then define the skew product $F : [0,1]^2 \to [0,1]^2$ by $F(x,y) := \big(Tx, g(x,y)\big)$. The attractor Λ of the map F is defined by $\Lambda := \bigcap_{n=0}^{\infty} F^n\big([0,1]^2\big)$. In a joint paper together with Franz Hofbauer and Károly Simon it is proved that the Hausdorff dimension of Λ equals $1 + \frac{\log \alpha}{-\log \lambda}$.
The result proved in this paper is a bit more general. It is not necessary that T is a tent map, and the modulus of the slope may be non-constant. Then one obtains that the Hausdorff dimension of the attractor equals $1 + \frac{h_{\mathrm{top}}(T)}{-\log \lambda}$.
Difficulties for proving this formula for the maps considered in this talk arise from two reasons. According to the assumptions on the map T there may be no finite Markov partition for T, and the map F may not be injective.
To overcome the problem that T need not be a Markov map the interval $[0,1]$ is "approximated" by sets D_k (usually these sets are Cantor sets) such that $T\big|_{D_k}$ is a Markov map, and $T\big|_{D_k}$ "converges" to T. Then the set $\Lambda_k := \bigcap_{n=0}^{\infty} F^n\big(D_k \times [0,1]\big)$ is considered, which is an "approximation" of Λ. In order to overcome the problem of the non-injectivity of F it is proved that the set of points in Λ having more than one pre-image under F^n for some n is "small". This is done using a "transversality condition".

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